Dynamic Economic Modelling
Problem Set 2

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Due date: 17:00PM, 19th of July 2023

Instructions: Attempt all questions and read them carefully. Be sure that your answers directly address the questions. You are encouraged to use a TEX language processor, or some other type setting program to write your answers. If submitting hand-written answers, then try to write neatly – grades will not be awarded for writing that is ineligible. It is not necessary to submit all mathematical/algebraic working out, but provide enough so that the marker can follow your intuition. Additionally, please submit your code in a clear to read format – I am happy with any method you choose to use, just make sure it is easy to read and that it looks professional.

Finally, I support and recommend you to work and collaborate with your peers in completing the problem set. If you're submitting your assignment as a group, please state members of the group clearly. Best of luck!

1 An analytic RBC model

Consider a version of the Real Business Cycle model presented in class in which there is 100% capital depreciation and utility is log-linear in consumption and leisure.

1. Discuss why the competitive equilibrium of this economy is equivalent to the following social planner problem:

$$\max_{\{c_t, l_t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t (\ln c_t - \chi l_t),$$

subject to:

$$\begin{aligned} y_t &= c_t + k_t, \\ y_t &= A_t k_{t-1}^{\alpha} l_t^{1-\alpha}, \\ \ln A_t &= \rho \ln A_t + \epsilon_t. \end{aligned}$$

Derive the intertemporal Euler equations for consumption and labour supply, and the intratemporal Euler labour-consumption for this economy.

2. ***__Bonus question__*** Define the output to consumption ratio $\vartheta_t = y_t/c_t$. Show that the Euler equation for consumption can then be written as a first-order ordinary difference equation in terms of ϑ_t and $E_t\vartheta_{t+1}$, and then calculate an expression for the steady-state value of ϑ_t. Plot a graph of the ordinary difference equation with ϑ_t on the horizontal axis and $E_t\vartheta_{t+1}$ on the vertical axis. Does your graph suggest that the ordinary difference equation for ϑ_t is stable?

   (a) if you think the ordinary difference equation is stable then comment what factors affect the speed with which ϑ_t converges to its steady-state value.

   (b) If you think the ordinary difference equation is unstable then comment on whether this is a sensible model.

   (c) How does the Blanchard-Kahn condition matter for whether the ordinary difference equation is stable or unstable? Is the transversality condition respected or violated in this model?

3. Assume that ϑ_t is equal to its steady state value, i.e., $\vartheta_t = \vartheta = y/c, \forall t$. Show that in this case the solution of the model is given by:

$$\begin{aligned} l_t &= \frac{1-\alpha}{\chi(1-\alpha\beta)}, \\ y_t &= A_t k_{t-1}^{\alpha} \left[\frac{1-\alpha}{\chi(1-\alpha\beta)}\right]^{1-\alpha}, \\ c_t &= (1-\alpha\beta) y_t, \\ k_t &= y_t - c_t \\ \ln A_t &= \rho \ln A_t + \epsilon_t. \end{aligned}$$

A calibration of the model is $\alpha = 0.4$, $\chi = 2$, $\beta = 0.99$, $\rho = 0.95$, and $\sigma_a = 0.01$. Derive the numerical equations corresponding to the second equation of y_t and calculate the steady state values of l, y, c, k, and A. Simulate the economy for 40 periods and calculate the standard deviation of simulated output, consumption, and labour supply.

Use impulse response functions to show how the endogenous variables in the economy react to a one standard deviation technology.

Hint: You can do this exercise in MATLAB using Dynare. If you use Dynare, then print out your code and the output produced, incluidng both calculations of standard deviations and a plot of the IRFs.

2 The RBC model

Consider a basic RBC model, where the social planner wants to maximise

$$\mathbb{E}_t \left[\sum_{i=0}^{\infty} \beta^i (U(C_{t+i}) - V(L_{t+i})) \right],$$

where C_t is consumption, L_t is hours worked, and β is the representative household's rate of time preference (discount factor). The economy faces constraints described by:

$$\begin{aligned} Y_t &= C_t + I_t, \\ Y_t &= F(K_t, L_t), \\ K_{t+1} &= I_t + (1-\delta)K_t, \end{aligned}$$

where $F(K_t, L_t)$ is the production technology of output, Y_t with constant returns to scale, I_t is investment, and δ is the rate of depreciation of capital. We can simplify the problem by combining the constraints into one equation:

$$F(K_t, L_t) = C_t + K_{t+1} - (1-\delta)K_t.$$

(a) Does the real business cycle (RBC) model predict that real wages should be procyclical or countercyclical? How about employment? Why?

(b) What does the empirical evidence say about the direction and magnitude of the fluctuations in these variables in comparison with the model's predictions?

(c) What are the implications of labour market developments for interpreting the validity of the RBC framework?