

Online Appendix: Global Determinacy According to HANK

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This Online Appendix provides the proofs of the propositions of “Global Determinacy According to HANK”, collected in Section **H**, together with the step-by-step algebraic derivations that support them. Section **A** derives the NKPC coefficient matrix C , the IS coefficient matrices A and B , and the auxiliary matrix Q . Section **B** computes the four Taylor-rule regime determinants and, in §**B.6**, derives the exact Blanchard-Kahn criterion for the unconstrained block, which the main text states without proof. Section **C** records the RANK and absorbing-state reductions. Section **D** derives the robust real rate rule’s piece determinants and enumerates its MSV solutions at $q = 1$. Section **E** derives the yield-curve-control determinants and carries the exhibits cited in Section **3** of the main text. Sections **F** and **G** treat the nominal and real long-term rate rules. Notation and abbreviations follow the main text, and GLM refers to the global invertibility theorem of [Gourieroux, Laffont, and Monfort \(1980\)](#); equation and figure numbers prefixed by a section letter refer to this appendix.

A Derivation of the Coefficient Matrices for the HANK Model

This section provides the explicit derivation of the coefficient matrices C , A , B , and Q that appear in the proofs of Section **H**. All notation follows the main paper.

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A.1 The NKPC coefficient matrix C

The NKPC in the two-state Markov formulation is

$$\pi = \beta K \pi + \kappa x, \quad (\text{A.1})$$

where the transition matrix is

$$K = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}.$$

Rearranging (A.1) yields

$$(I - \beta K)\pi = \kappa x \implies \pi = \kappa(I - \beta K)^{-1}x \equiv Cx. \quad (\text{A.2})$$

Computing the inverse. We have

$$I - \beta K = \begin{pmatrix} 1 - \beta p & -\beta(1-p) \\ -\beta(1-q) & 1 - \beta q \end{pmatrix}.$$

The determinant is

$$D_\beta \equiv \det(I - \beta K) = (1 - \beta p)(1 - \beta q) - \beta^2(1-p)(1-q). \quad (\text{A.3})$$

Expanding:

$$\begin{aligned} D_\beta &= 1 - \beta p - \beta q + \beta^2 p q - \beta^2(1-p-q+pq) \\ &= 1 - \beta p - \beta q + \beta^2 p q - \beta^2 + \beta^2 p + \beta^2 q - \beta^2 p q \\ &= 1 - \beta(p+q) + \beta^2(p+q) - \beta^2 \\ &= (1 - \beta)[1 - \beta(p+q-1)]. \end{aligned}$$

Since $0 \leq \beta < 1$ and $p+q-1 \leq 1$, we have $1 - \beta(p+q-1) \geq 1 - \beta > 0$, so $D_\beta > 0$ for every $(p, q) \in [0, 1]^2$. Hence the inverse exists and is given by the standard 2×2 formula:

$$(I - \beta K)^{-1} = \frac{1}{D_\beta} \begin{pmatrix} 1 - \beta q & \beta(1-p) \\ \beta(1-q) & 1 - \beta p \end{pmatrix}. \quad (\text{A.4})$$

Therefore, the NKPC coefficient matrix is

$$C = \frac{\kappa}{D_\beta} \begin{pmatrix} 1 - \beta q & \beta(1 - p) \\ \beta(1 - q) & 1 - \beta p \end{pmatrix}. \quad (\text{A.5})$$

Lemma 1 (Commutativity) *The matrices K and $(I - \beta K)^{-1}$ commute, and hence $KC = CK$.*

Proof. Since $(I - \beta K)^{-1}$ is a polynomial in K (by the Cayley–Hamilton theorem for 2×2 matrices, $(I - \beta K)^{-1} = \frac{1}{D_\beta} [\text{tr}(I - \beta K)I - (I - \beta K)]$, which is a linear combination of I and K), it commutes with K . More directly, one can verify by explicit multiplication that

$$K \cdot (I - \beta K)^{-1} = (I - \beta K)^{-1} \cdot K.$$

Both sides yield

$$\frac{1}{D_\beta} \begin{pmatrix} p(1 - \beta q) + \beta(1 - p)(1 - q) & (1 - p)(1 - \beta p) + \beta p(1 - p) \\ (1 - q)(1 - \beta q) + \beta q(1 - q) & q(1 - \beta p) + \beta(1 - p)(1 - q) \end{pmatrix}. \quad \blacksquare$$

A.2 The IS coefficient matrices A and B

The reduced IS equation, after substituting the NKPC and Taylor rule into the DISE, is

$$Ax + \widehat{\sigma} \max\{-\mu\iota, Bx\} = \varepsilon, \quad (\text{A.6})$$

where

$$A = (I - \delta K) - \widehat{\sigma} KC, \quad B = \phi C. \quad (\text{A.7})$$

The matrix $I - \delta K$.

$$I - \delta K = \begin{pmatrix} 1 - \delta p & -\delta(1 - p) \\ -\delta(1 - q) & 1 - \delta q \end{pmatrix}. \quad (\text{A.8})$$

The matrix $\widehat{\sigma} KC$. Using $KC = CK$ from Lemma 1 and equation (A.5):

$$KC = \frac{\kappa}{D_\beta} K \begin{pmatrix} 1 - \beta q & \beta(1 - p) \\ \beta(1 - q) & 1 - \beta p \end{pmatrix}. \quad (\text{A.9})$$

Computing:

$$\begin{aligned}
(KC)_{11} &= \frac{\kappa}{D_\beta} [p(1 - \beta q) + (1 - p)\beta(1 - q)] = \frac{\kappa}{D_\beta} [p - \beta q p + \beta(1 - q) - \beta p(1 - q)] \\
&= \frac{\kappa}{D_\beta} [p + \beta(1 - q)(1 - p) - \beta p q], \\
(KC)_{12} &= \frac{\kappa}{D_\beta} [p\beta(1 - p) + (1 - p)(1 - \beta p)] = \frac{\kappa(1 - p)}{D_\beta} [\beta p + 1 - \beta p] = \frac{\kappa(1 - p)}{D_\beta}, \\
(KC)_{21} &= \frac{\kappa}{D_\beta} [(1 - q)(1 - \beta q) + q\beta(1 - q)] = \frac{\kappa(1 - q)}{D_\beta} [1 - \beta q + \beta q] = \frac{\kappa(1 - q)}{D_\beta}, \\
(KC)_{22} &= \frac{\kappa}{D_\beta} [(1 - q)\beta(1 - p) + q(1 - \beta p)] = \frac{\kappa}{D_\beta} [q + \beta(1 - p)(1 - q) - \beta p q].
\end{aligned}$$

The matrix A. Combining (A.8) and (A.9):

$$A = \begin{pmatrix} 1 - \delta p - \frac{\widehat{\sigma}\kappa}{D_\beta} [p + \beta(1 - p)(1 - q) - \beta p q] & -\delta(1 - p) - \frac{\widehat{\sigma}\kappa(1 - p)}{D_\beta} \\ -\delta(1 - q) - \frac{\widehat{\sigma}\kappa(1 - q)}{D_\beta} & 1 - \delta q - \frac{\widehat{\sigma}\kappa}{D_\beta} [q + \beta(1 - p)(1 - q) - \beta p q] \end{pmatrix}. \quad (\text{A.10})$$

The matrix B.

$$B = \phi C = \frac{\kappa\phi}{D_\beta} \begin{pmatrix} 1 - \beta q & \beta(1 - p) \\ \beta(1 - q) & 1 - \beta p \end{pmatrix}. \quad (\text{A.11})$$

A.3 The Q matrix

The key computational device is to multiply each regime matrix $M_s = A + \widehat{\sigma}H_s B$ by $(I - \beta K)$ on the right, using the identity $C(I - \beta K) = \kappa I$. Define

$$Q \equiv A(I - \beta K) = (I - \delta K)(I - \beta K) - \kappa \widehat{\sigma} K. \quad (\text{A.12})$$

The second equality follows because $\widehat{\sigma} K C \cdot (I - \beta K) = \widehat{\sigma} K \cdot \kappa I = \kappa \widehat{\sigma} K$.

Element-by-element expansion. We compute $(I - \delta K)(I - \beta K)$ first. Denote this product by P :

$$P = \begin{pmatrix} 1 - \delta p & -\delta(1 - p) \\ -\delta(1 - q) & 1 - \delta q \end{pmatrix} \begin{pmatrix} 1 - \beta p & -\beta(1 - p) \\ -\beta(1 - q) & 1 - \beta q \end{pmatrix}.$$

Entry P_{11} :

$$P_{11} = (1 - \delta p)(1 - \beta p) + \delta \beta (1 - p)(1 - q).$$

Entry P_{12} :

$$\begin{aligned} P_{12} &= -(1 - \delta p)\beta(1 - p) - \delta(1 - p)(1 - \beta q) \\ &= -(1 - p)[\beta(1 - \delta p) + \delta(1 - \beta q)] \\ &= -(1 - p)[\beta + \delta - 2\beta\delta p - \delta\beta q + \delta\beta p] \\ &= -(1 - p)[\beta + \delta - \beta\delta p - \delta\beta q] \\ &= -(1 - p)[\beta(1 - \delta q) + \delta(1 - \beta p)]. \end{aligned}$$

Entry P_{21} :

$$\begin{aligned} P_{21} &= -\delta(1 - q)(1 - \beta p) - (1 - \delta q)\beta(1 - q) \\ &= -(1 - q)[\delta(1 - \beta p) + \beta(1 - \delta q)]. \end{aligned}$$

Note the symmetry: P_{21} is obtained from P_{12} by swapping $p \leftrightarrow q$.

Entry P_{22} :

$$P_{22} = \delta\beta(1 - q)(1 - p) + (1 - \delta q)(1 - \beta q).$$

Now subtracting $\kappa\widehat{\sigma}\mathbf{K}$:

$$\mathbf{Q} = \mathbf{P} - \kappa\widehat{\sigma}\mathbf{K} = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}, \tag{A.13}$$

where

$$Q_{11} = (1 - \delta p)(1 - \beta p) + \delta \beta(1 - p)(1 - q) - \kappa \widehat{\sigma} p, \quad (\text{A.14})$$

$$\begin{aligned} Q_{12} &= -(1 - p) [\beta(1 - \delta q) + \delta(1 - \beta p)] - \kappa \widehat{\sigma}(1 - p) \\ &= -(1 - p) [\beta(1 - \delta q) + \delta(1 - \beta p) + \kappa \widehat{\sigma}], \end{aligned} \quad (\text{A.15})$$

$$Q_{21} = -(1 - q) [\delta(1 - \beta p) + \beta(1 - \delta q) + \kappa \widehat{\sigma}], \quad (\text{A.16})$$

$$Q_{22} = (1 - \delta q)(1 - \beta q) + \delta \beta(1 - p)(1 - q) - \kappa \widehat{\sigma} q. \quad (\text{A.17})$$

Off-diagonal structure. Notice that

$$\frac{Q_{12}}{Q_{21}} = \frac{1 - p}{1 - q}, \quad (\text{A.18})$$

which holds whenever $Q_{21} \neq 0$. Defining the common factor

$$\gamma \equiv \beta(1 - \delta q) + \delta(1 - \beta p) + \kappa \widehat{\sigma} = \beta + \delta - \beta \delta(p + q) + \kappa \widehat{\sigma}, \quad (\text{A.19})$$

we can write $Q_{12} = -(1 - p)\gamma$ and $Q_{21} = -(1 - q)\gamma$.

Alternative form for diagonal entries. It is useful to rewrite the diagonal entries as follows. For Q_{11} , a compact rewriting uses the identity $(1 - \delta p)(1 - \beta p) = (1 - \delta)(1 - \beta p) + \delta(1 - p)(1 - \beta p)$:

$$Q_{11} = (1 - \delta)(1 - \beta p) + \delta(1 - p) [(1 - \beta p) + \beta(1 - q)] - \kappa \widehat{\sigma} p. \quad (\text{A.20})$$

The most useful form remains the original (A.14).

Similarly for Q_{22} :

$$Q_{22} = (1 - \delta)(1 - \beta q) + \delta(1 - q) [(1 - \beta q) + \beta(1 - p)] - \kappa \widehat{\sigma} q, \quad (\text{A.21})$$

which simplifies to the original expression (A.17).

B Step-by-Step Determinant Computations (Taylor Rule)

We compute the four regime determinants $\det[\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{H}_s]$ for $s \in \{PP, PZ, ZP, ZZ\}$. Recall from §H.1 that $\det \mathbf{M}_s \cdot D_\beta = \det[\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{H}_s]$, where $D_\beta > 0$.

B.1 $\det(\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{I})$ – the PP case

When the policy rate is positive in both states, $\mathbf{H}_{PP} = \mathbf{I}$, and we need

$$\det \mathbf{M}_{PP} \cdot D_\beta = \det(\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{I}). \quad (\text{B.1})$$

The matrix is

$$\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{I} = \begin{pmatrix} Q_{11} + \kappa\widehat{\sigma}\phi & Q_{12} \\ Q_{21} & Q_{22} + \kappa\widehat{\sigma}\phi \end{pmatrix}.$$

Step 1: Substitute diagonal entries. Using (A.14) and (A.17):

$$\begin{aligned} Q_{11} + \kappa\widehat{\sigma}\phi &= (1 - \delta p)(1 - \beta p) + \delta\beta(1 - p)(1 - q) + \kappa\widehat{\sigma}(\phi - p), \\ Q_{22} + \kappa\widehat{\sigma}\phi &= (1 - \delta q)(1 - \beta q) + \delta\beta(1 - p)(1 - q) + \kappa\widehat{\sigma}(\phi - q). \end{aligned}$$

Step 2: Compute the determinant.

$$\det = [Q_{11} + \kappa\widehat{\sigma}\phi][Q_{22} + \kappa\widehat{\sigma}\phi] - Q_{12}Q_{21}. \quad (\text{B.2})$$

Using $Q_{12} = -(1 - p)\gamma$ and $Q_{21} = -(1 - q)\gamma$:

$$Q_{12}Q_{21} = (1 - p)(1 - q)\gamma^2. \quad (\text{B.3})$$

Step 3: Factor. Expanding (B.2) term by term is laborious; the factorisation follows directly from the spectral structure of $\mathbf{K}$. Let $\rho \equiv p + q - 1$. The matrix $\mathbf{K}$ has eigenvalues 1 and ρ , with eigenvectors $\boldsymbol{\iota} = (1, 1)^\top$ and $(1 - p, -(1 - q))^\top$. By (A.12), $\mathbf{Q}$ is a polynomial in $\mathbf{K}$, so it shares these eigenvectors, with eigenvalues

$$(1 - \delta)(1 - \beta) - \kappa\widehat{\sigma} \equiv H \quad \text{and} \quad (1 - \delta\rho)(1 - \beta\rho) - \kappa\widehat{\sigma}\rho = -\kappa\widehat{\sigma}\vartheta,$$

where H is as in the main text and the second equality uses (B.16), which can be written compactly as $\vartheta = \rho - (1 - \delta\rho)(1 - \beta\rho)/(\kappa\widehat{\sigma})$, since $(1 - \delta) + \delta(2 - p - q) = 1 - \delta\rho$ and $1 - \beta q - \beta p + \beta = 1 - \beta\rho$. Adding $\kappa\widehat{\sigma}\phi\mathbf{I}$ shifts both eigenvalues by $\kappa\widehat{\sigma}\phi$, so $\det(\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{I}) = (H + \kappa\widehat{\sigma}\phi)\kappa\widehat{\sigma}(\phi - \vartheta)$, that is,

$$\boxed{\det \mathbf{M}_{PP} \cdot D_\beta = \kappa\widehat{\sigma}[(1 - \delta)(1 - \beta) + \kappa\widehat{\sigma}(\phi - 1)](\phi - \vartheta).} \quad (\text{B.4})$$

The same argument gives the two identities used below. The second row of $\mathbf{Q}\iota = H\iota$ reads $Q_{21} + Q_{22} = H$, so $Q_{22} = H + (1 - q)\gamma$; and the trace of $\mathbf{Q}$ is the sum of its eigenvalues, $Q_{11} + Q_{22} = H - \kappa\widehat{\sigma}\vartheta$, so $Q_{11} = -\kappa\widehat{\sigma}\vartheta - (1 - q)\gamma$.

B.2 $\det(\mathbf{Q})$ – the ZZ case

When the policy rate is at the ELB in both states, $H_{ZZ} = \mathbf{0}$:

$$\det \mathbf{M}_{ZZ} \cdot D_\beta = \det(\mathbf{Q}). \quad (\text{B.5})$$

Step 1: Write out the determinant.

$$\det(\mathbf{Q}) = Q_{11}Q_{22} - Q_{12}Q_{21} = Q_{11}Q_{22} - (1 - p)(1 - q)\gamma^2. \quad (\text{B.6})$$

Step 2: Expand $Q_{11}Q_{22}$.

$$\begin{aligned} Q_{11}Q_{22} &= [(1 - \delta p)(1 - \beta p) + \delta\beta(1 - p)(1 - q) - \kappa\widehat{\sigma}p] \\ &\quad \times [(1 - \delta q)(1 - \beta q) + \delta\beta(1 - p)(1 - q) - \kappa\widehat{\sigma}q]. \end{aligned}$$

This is the PP determinant evaluated at $\phi = 0$. From (B.4) with $\phi = 0$:

$$\det(\mathbf{Q}) = \kappa\widehat{\sigma}[(1 - \delta)(1 - \beta) - \kappa\widehat{\sigma}](0 - \vartheta).$$

Hence:

$$\boxed{\det \mathbf{M}_{ZZ} \cdot D_\beta = [-(\kappa\widehat{\sigma})^2 + \kappa\widehat{\sigma}(1 - \delta)(1 - \beta)](-\vartheta).} \quad (\text{B.7})$$

B.3 $\det(\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{e}_1\mathbf{e}_1^\top)$ – the PZ case

When the policy rate is positive in state 1 only, $\mathbf{H}_{PZ} = \mathbf{e}_1\mathbf{e}_1^\top$:

$$\det \mathbf{M}_{PZ} \cdot D_\beta = \det(\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{e}_1\mathbf{e}_1^\top). \quad (\text{B.8})$$

The matrix is

$$\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{e}_1\mathbf{e}_1^\top = \begin{pmatrix} Q_{11} + \kappa\widehat{\sigma}\phi & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}.$$

Step 1: Expand along the updated entry. The determinant is linear in the $(1, 1)$ entry, so expanding along it gives, with no invertibility of $\mathbf{Q}$ required,

$$\det(\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{e}_1\mathbf{e}_1^\top) = \det(\mathbf{Q}) + \kappa\widehat{\sigma}\phi Q_{22}. \quad (\text{B.9})$$

Step 2: Substitute. From (B.7) and (A.17):

$$\begin{aligned} \det \mathbf{M}_{PZ} \cdot D_\beta &= [-(\kappa\widehat{\sigma})^2 + \kappa\widehat{\sigma}(1-\delta)(1-\beta)](-\vartheta) \\ &\quad + \kappa\widehat{\sigma}\phi[(1-\delta q)(1-\beta q) + \delta\beta(1-p)(1-q) - \kappa\widehat{\sigma}q]. \end{aligned} \quad (\text{B.10})$$

Step 3: Rearrange. Substituting $Q_{22} = H + (1-q)\gamma$ from §B.1 into (B.9), with $\det \mathbf{Q} = \kappa\widehat{\sigma}H(-\vartheta)$ from (B.7), gives $\kappa\widehat{\sigma}H(\phi - \vartheta) + \kappa\widehat{\sigma}\phi(1-q)\gamma$. Since $\kappa\widehat{\sigma}H = \kappa\widehat{\sigma}(1-\delta)(1-\beta) - (\kappa\widehat{\sigma})^2$, and the bracket in the remainder below is γ of (A.19), the result is:

$$\begin{aligned} \det \mathbf{M}_{PZ} \cdot D_\beta &= [\kappa\widehat{\sigma}(1-\delta)(1-\beta) - (\kappa\widehat{\sigma})^2](\phi - \vartheta) \\ &\quad + \kappa\widehat{\sigma}\phi(1-q)[\beta(1-\delta) + \delta(1-\beta p - \beta q + \beta) + \kappa\widehat{\sigma}]. \end{aligned}$$

(B.11)

Verification. Setting $\phi = 0$ in (B.11) gives $[\kappa\widehat{\sigma}(1-\delta)(1-\beta) - (\kappa\widehat{\sigma})^2](\phi - \vartheta)$, which is $\det \mathbf{Q}$ by (B.7); the remainder carries a factor ϕ and vanishes. Setting $q = 1$ eliminates the remainder, giving the clean factorisation

$$\det \mathbf{M}_{PZ} \cdot D_\beta|_{q=1} = [\kappa\widehat{\sigma}(1-\delta)(1-\beta) - (\kappa\widehat{\sigma})^2](\phi - \vartheta).$$

B.4 $\det(\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{e}_2\mathbf{e}_2^\top)$ – the ZP case

When the policy rate is positive in state 2 only, $\mathbf{H}_{ZP} = \mathbf{e}_2\mathbf{e}_2^\top$:

$$\det \mathbf{M}_{ZP} \cdot D_\beta = \det(\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{e}_2\mathbf{e}_2^\top). \quad (\text{B.12})$$

Step 1: Expand along the updated entry. As for PZ, expanding along the (2, 2) entry gives

$$\det(\mathbf{Q} + \kappa\widehat{\sigma}\phi\mathbf{e}_2\mathbf{e}_2^\top) = \det(\mathbf{Q}) + \kappa\widehat{\sigma}\phi Q_{11}. \quad (\text{B.13})$$

Step 2: Substitute. From (B.7) and (A.14):

$$\begin{aligned} \det \mathbf{M}_{ZP} \cdot D_\beta &= [-(\kappa\widehat{\sigma})^2 + \kappa\widehat{\sigma}(1-\delta)(1-\beta)](-\vartheta) \\ &\quad + \kappa\widehat{\sigma}\phi[(1-\delta p)(1-\beta p) + \delta\beta(1-p)(1-q) - \kappa\widehat{\sigma}p]. \end{aligned} \quad (\text{B.14})$$

Step 3: Rearrange. Substituting $Q_{11} = -\kappa\widehat{\sigma}\vartheta - (1-q)\gamma$ from §B.1 into (B.13) gives $\kappa\widehat{\sigma}(H + \kappa\widehat{\sigma}\phi)(-\vartheta) - \kappa\widehat{\sigma}\phi(1-q)\gamma$. Since $\kappa\widehat{\sigma}(H + \kappa\widehat{\sigma}\phi) = (\kappa\widehat{\sigma})^2(\phi - 1) + \kappa\widehat{\sigma}(1-\delta)(1-\beta)$, and the bracket in the remainder below is again γ :

$$\boxed{\begin{aligned} \det \mathbf{M}_{ZP} \cdot D_\beta &= [(\kappa\widehat{\sigma})^2(\phi - 1) + \kappa\widehat{\sigma}(1-\delta)(1-\beta)](-\vartheta) \\ &\quad - \kappa\widehat{\sigma}\phi(1-q)[\delta(1-\beta p - \beta q + \beta) + \kappa\widehat{\sigma} + \beta(1-\delta)]. \end{aligned}} \quad (\text{B.15})$$

Verification. Setting $\phi = 0$ recovers $\det \mathbf{Q}$ from (B.7). Setting $q = 1$ eliminates the remainder:

$$\det \mathbf{M}_{ZP} \cdot D_\beta|_{q=1} = [(\kappa\widehat{\sigma})^2(\phi - 1) + \kappa\widehat{\sigma}(1-\delta)(1-\beta)](-\vartheta).$$

B.5 Definition and properties of ϑ

The quantity ϑ is defined by the eigenvalue $-\kappa\widehat{\sigma}\vartheta$ of $\mathbf{Q}$ associated with the eigenvalue $\rho = p + q - 1$ of $\mathbf{K}$ (§B.1). From (B.4), $\det \mathbf{M}_{PP} \cdot D_\beta = 0$ when either $\phi = 1 - (1-\delta)(1-\beta)/(\kappa\widehat{\sigma})$ (from the middle factor) or $\phi = \vartheta$ (from the last factor), the latter being the only root that depends on the transition probabilities p and q . Explicitly,

$$\vartheta \equiv p + q - 1 - \frac{[(1-\delta) + \delta(2-p-q)](1-\beta q - \beta p + \beta)}{\kappa\widehat{\sigma}}. \quad (\text{B.16})$$

Simplification for $q = 1$. Setting $q = 1$:

$$\begin{aligned}\vartheta|_{q=1} &= p + 1 - 1 - \frac{[(1 - \delta) + \delta(2 - p - 1)](1 - \beta - \beta p + \beta)}{\kappa \widehat{\sigma}} \\ &= p - \frac{(1 - \delta + \delta(1 - p))(1 - \beta p)}{\kappa \widehat{\sigma}} \\ &= p - \frac{(1 - \delta p)(1 - \beta p)}{\kappa \widehat{\sigma}}.\end{aligned}$$

This confirms the expression used in §H.1:

$$\vartheta|_{q=1} = p - \frac{(1 - \delta p)(1 - \beta p)}{\kappa \widehat{\sigma}}. \quad (\text{B.17})$$

B.6 Local determinacy: the exact Blanchard-Kahn criterion

The four regime determinants of §B.1–§B.4 govern the *uniqueness* leg of global determinacy. This subsection derives the other leg – local determinacy of the unconstrained block – and so supplies in full the criterion that Section 2.1 of the main text states without proof. Neither the Markov kernel K nor the effective lower bound enters anywhere below: local determinacy is a property of the unconstrained two-equation system alone, which is why the resulting bounds are free of p and q .

The unconstrained block. Away from the lower bound the rule is $i_t = \phi \pi_t$. Substituting it into the DISE and stacking the DISE and the NKPC in $z_t \equiv (x_t, \pi_t)^\top$ gives the linear pencil

$$N_0 z_t = N_1 \mathbb{E}_t z_{t+1}, \quad N_0 \equiv \begin{bmatrix} 1 & \widehat{\sigma} \phi \\ -\kappa & 1 \end{bmatrix}, \quad N_1 \equiv \begin{bmatrix} \delta & \widehat{\sigma} \\ 0 & \beta \end{bmatrix}, \quad D \equiv \det N_0 = 1 + \widehat{\sigma} \kappa \phi. \quad (\text{B.18})$$

Both x_t and π_t are jump variables and the system carries no predetermined state, so the Blanchard-Kahn condition is that both generalised eigenvalues of (B.18) – the roots of $\det(N_0 - z N_1) = 0$ – lie strictly outside the unit circle. Whenever $D \neq 0$ that is equivalent to both eigenvalues of the backward map

$$\Psi \equiv N_0^{-1} N_1 = \frac{1}{D} \begin{bmatrix} \delta & \widehat{\sigma}(1 - \beta \phi) \\ \kappa \delta & \kappa \widehat{\sigma} + \beta \end{bmatrix}, \quad \text{tr } \Psi = \frac{\delta + \beta + \kappa \widehat{\sigma}}{D}, \quad \det \Psi = \frac{\delta \beta}{D}, \quad (\text{B.19})$$

lying strictly inside it. Working with Ψ rather than with the forward map $N_1^{-1}N_0$ costs nothing and buys the degenerate case $\delta\beta = 0$, at which the forward map has an infinite eigenvalue while Ψ merely has a zero one.

The three Schur-Cohn conditions. Let $p_\Psi(z) \equiv z^2 - (\text{tr } \Psi)z + \det \Psi = (z - z_1)(z - z_2)$ denote the characteristic polynomial of Ψ . Both of its roots lie strictly inside the unit circle if and only if the three Schur-Cohn conditions

$$p_\Psi(1) > 0, \quad p_\Psi(-1) > 0, \quad |\det \Psi| < 1, \quad (\text{B.20})$$

hold. The equivalence is elementary for a real quadratic. *Necessity:* $|\det \Psi| = |z_1 z_2| < 1$, and $p_\Psi(\pm 1) = (1 \mp z_1)(1 \mp z_2)$ is a product of two positive reals when the roots are real and equals $|1 \mp z_1|^2 > 0$ when they are a complex-conjugate pair. *Sufficiency:* for a conjugate pair $|\det \Psi| = |z_1|^2 < 1$ is already the claim. For real roots p_Ψ is an upward parabola, so $p_\Psi(1) > 0$ places both roots on the same side of 1 and $p_\Psi(-1) > 0$ places both on the same side of -1 ; of the configurations other than the claim, “both above 1” and “both below -1 ” each give $z_1 z_2 > 1$, contradicting $|\det \Psi| < 1$, and “both above 1 and both below -1 ” is empty. Hence both roots lie in $(-1, 1)$.

The criterion in the notation of the main text. Multiplying the first two conditions of (B.20) through by D and substituting the trace and determinant of (B.19),

$$D p_\Psi(1) = D - (\delta + \beta + \kappa\widehat{\sigma}) + \delta\beta = (1 - \delta)(1 - \beta) + \kappa\widehat{\sigma}(\phi - 1) = F, \quad (\text{B.21})$$

$$D p_\Psi(-1) = D + (\delta + \beta + \kappa\widehat{\sigma}) + \delta\beta = (1 + \delta)(1 + \beta) + \kappa\widehat{\sigma}(\phi + 1) = G, \quad (\text{B.22})$$

with F and G the quantities of those names in Section 2.1 of the main text. The letter F is shared, and not by accident, with the proof of Proposition 1, where $\det \mathbf{M}_{PP} \cdot D_\beta = \kappa\widehat{\sigma} F(\phi - \vartheta)$: the trace-root factor of local determinacy and the unit-eigenvalue factor of the PP determinant are literally the same linear function of ϕ . Since $D \neq 0$, $p_\Psi(1) > 0$ says that F and D carry a common sign and $p_\Psi(-1) > 0$ that G and D do, while $|\det \Psi| < 1$ reads $|\delta\beta| < |D|$. Collecting the three,

$$\text{local determinacy} \iff \text{sgn } F = \text{sgn } G = \text{sgn } D \quad \text{and} \quad |\delta\beta| < |D|, \quad (\text{B.23})$$

which is the criterion of the main text. A common sign is by construction a nonzero sign, so (B.23) carries the invertibility of N_0 with it and needs no separate proviso.

Which condition is which displayed bound. F is linear in ϕ with slope $\kappa\widehat{\sigma}$ and vanishes at $\phi = 1 - (1 - \delta)(1 - \beta)/(\kappa\widehat{\sigma})$, which is ϕ_{low}^* when $\widehat{\sigma} > 0$ and ϕ_1^* when $\widehat{\sigma} < 0$ – one expression written twice, with the sign of $\widehat{\sigma}$ absorbed into $|\widehat{\sigma}|$. Because the slope carries the sign of $\widehat{\sigma}$, $F > 0$ reads $\phi > \phi_{\text{low}}^*$ under standard aggregate demand and $\phi < \phi_1^*$ under IADL: the direction of the Taylor principle flips with the demand channel, and this is the condition the main text calls the *trace-root* condition. The determinant condition $|\delta\beta| < |D|$ is what it calls the *determinant-root* condition. For $\widehat{\sigma} > 0$, $\delta > -1$, and $\phi \geq 0$ the condition on G is automatic, since $G > 0$ outright; the common sign is then positive, and $D > |\delta\beta| = \delta\beta$ for $\delta \geq 0$ is exactly $\phi > \phi_a \equiv (\delta\beta - 1)/(\widehat{\sigma}\kappa)$, so the two binding conditions collapse into the displayed $\phi > \max\{\phi_{\text{low}}^*, \phi_a\}$. Under IADL no such collapse is available. There $D = 1 - \kappa|\widehat{\sigma}|\phi$ is decreasing in ϕ and $|\delta\beta| < |D|$ splits into two disjoint intervals – $\phi < (1 - |\delta\beta|)/(\kappa|\widehat{\sigma}|)$, on which $D > 0$, and $\phi > (1 + |\delta\beta|)/(\kappa|\widehat{\sigma}|)$, on which $D < 0$ – which carry opposite common signs and cannot be merged into a single bound on ϕ . That, and the fact that G is no longer automatically signed once $\widehat{\sigma} < 0$, is why the main text states the IADL criterion in the form (B.23) rather than as a second displayed inequality.

Numerical verification. The Blanchard-Kahn leg of the shading of Figure 1 is produced by counting the eigenvalues of the pencil (B.18) directly, point by point, rather than by evaluating (B.23) (Julia/generate_figure1_tr.jl in the replication package), so the figure and the criterion are independent computations of the same set. The frozen independent certification also reproduces the closed forms for $\text{tr } \Psi$ and $\det \Psi$ against a direct solve of the pencil with maximum relative errors of 4.5×10^{-12} and 1.7×10^{-11} , respectively. The Schur-Cohn conditions and the $F/G/D$ criterion each have zero disagreements with the generalised-eigenvalue oracle on 120,000 random draws, and the criterion has zero disagreements on all 425,641 points of a dense 1400×400 grid over the IADL region of Figure 1 ($\phi = 0.5$, $\lambda \in [0.3, 0.9995]$, $\chi \in [1.005, 4]$).

C Special Case Reductions

C.1 RANK limit ($\delta = 1, \widehat{\sigma} = 1/\sigma$)

In the representative agent New Keynesian (RANK) model, there are no restricted households, so $\lambda = 0$; setting $\chi = 1$ as well gives $\delta = 1$. From the expressions for δ and $\widehat{\sigma}$ in the main text:

$$\delta = 1 + \frac{(\chi - 1)(1 - s)}{1 - \lambda\chi} = 1, \quad \widehat{\sigma} = \frac{1 - \lambda}{\sigma(1 - \lambda\chi)} = \frac{1}{\sigma}.$$

PP determinant. From (B.4) with $\delta = 1$:

$$\det M_{PP} \cdot D_\beta = \frac{\kappa}{\sigma} \left[(1 - 1)(1 - \beta) + \frac{\kappa}{\sigma}(\phi - 1) \right] (\phi - \vartheta) = \frac{\kappa^2}{\sigma^2}(\phi - 1)(\phi - \vartheta).$$

This is the factored form for the RANK model's PP determinant (cf. [Murakami, Shchapov, and Zhang \(2026\)](#), without fiscal policy).

ZZ determinant. From (B.7) with $\delta = 1$:

$$\det M_{ZZ} \cdot D_\beta = \left[-\frac{\kappa^2}{\sigma^2} + 0 \right] (-\vartheta) = \frac{\kappa^2}{\sigma^2} \vartheta.$$

ϑ in the RANK limit. From (B.16) with $\delta = 1$:

$$\vartheta|_{\delta=1} = p + q - 1 - \frac{(2 - p - q)(1 - \beta q - \beta p + \beta)}{\kappa/\sigma}. \quad (\text{C.1})$$

With $q = 1$:

$$\vartheta|_{\delta=1, q=1} = p - \frac{(1 - p)(1 - \beta p)}{\kappa/\sigma} = p - \frac{\sigma(1 - p)(1 - \beta p)}{\kappa},$$

which matches the standard RANK expression.

Sign conditions. With $F \equiv (1 - \delta)(1 - \beta) + \kappa\widehat{\sigma}(\phi - 1)$ and $H \equiv (1 - \delta)(1 - \beta) - \kappa\widehat{\sigma}$ as in the proof of Proposition 1, in the RANK limit $H = -\kappa\widehat{\sigma} < 0$, while $F = \kappa\widehat{\sigma}(\phi - 1)$ carries the sign of $\phi - 1$. For an active rule ($\phi > 1$), $F > 0$, so $F \cdot H < 0$ and the same-sign condition necessarily fails – the four determinants display the mixed pattern $\{+, -, -, +\}$ (in the order PP, PZ, ZP, ZZ) whenever $0 < \vartheta < \phi$. By the reduction (H.15), $F \cdot H > 0$ and $(\phi - \vartheta)(-\vartheta) > 0$, uniqueness in the RANK limit requires *passive* policy, $\phi < 1$, together with

$\vartheta < 0$ or $\vartheta > \phi$ – consistent with the abstract and with [Ascari and Mavroeidis \(2022\)](#) and [Murakami, Shchapov, and Zhang \(2026\)](#). At $q = 1$, $\vartheta < 0$ obtains for p below a threshold ($p^* \approx 0.75$ at $\sigma = 2$), recovering the standard result that a unique MSV solution requires p to be sufficiently small ($p\Lambda_0 < 1$, equivalently $\vartheta < 0$). Since passive policy violates the Blanchard-Kahn conditions, the RANK model is not globally determinate at any ϕ .

C.2 Absorbing state ($q = 1$)

Setting $q = 1$ (the second state is absorbing, i.e. the PIR steady state), the transition matrix becomes

$$K|_{q=1} = \begin{pmatrix} p & 1-p \\ 0 & 1 \end{pmatrix}.$$

Simplification of γ . From (A.19):

$$\gamma|_{q=1} = \beta(1 - \delta) + \delta(1 - \beta p) + \kappa\widehat{\sigma} = \beta + \delta - \beta\delta(p + 1) + \kappa\widehat{\sigma} = \beta + \delta - \beta\delta p - \beta\delta + \kappa\widehat{\sigma}.$$

Since $q = 1$ appears in the remainder terms of the PZ and ZP determinants as $(1 - q)$, these terms vanish identically.

PP determinant ($q = 1$). From (B.4):

$$\det M_{PP} \cdot D_\beta|_{q=1} = \kappa\widehat{\sigma}[(1 - \delta)(1 - \beta) + \kappa\widehat{\sigma}(\phi - 1)] \left(\phi - p + \frac{(1 - \delta p)(1 - \beta p)}{\kappa\widehat{\sigma}} \right). \quad (C.2)$$

ZZ determinant ($q = 1$). From (B.7):

$$\det M_{ZZ} \cdot D_\beta|_{q=1} = [-(\kappa\widehat{\sigma})^2 + \kappa\widehat{\sigma}(1 - \delta)(1 - \beta)] \left(-p + \frac{(1 - \delta p)(1 - \beta p)}{\kappa\widehat{\sigma}} \right). \quad (C.3)$$

PZ determinant ($q = 1$). The $(1 - q)$ remainder in (B.11) vanishes:

$$\det M_{PZ} \cdot D_\beta|_{q=1} = [\kappa\widehat{\sigma}(1 - \delta)(1 - \beta) - (\kappa\widehat{\sigma})^2](\phi - \vartheta)|_{q=1}. \quad (C.4)$$

ZP determinant ($q = 1$). The $(1 - q)$ remainder in (B.15) vanishes:

$$\det M_{ZP} \cdot D_\beta|_{q=1} = [(\kappa\widehat{\sigma})^2(\phi - 1) + \kappa\widehat{\sigma}(1 - \delta)(1 - \beta)](-\vartheta)|_{q=1}. \quad (C.5)$$

These coincide with the Taylor-rule expressions (H.10)–(H.13) of §H.1.

D Robust Real Rate Rule Determinants

This section derives the canonical – continuity-valid – piecewise-linear formulation of the HANK model under the Robust Real Rate rule (RR), $i_t = \max\{-\mu, r_t + \phi\pi_t\}$ with $r_t \equiv i_t - \mathbb{E}_t\pi_{t+1}$, computes its four piece-Jacobian determinants, and solves the four regime configurations in closed form at $q = 1$, as used in the proof of the RR proposition in §H.2. It also records why a stacked regime-by-regime construction is not a valid input to the GLM theorem under this rule.

D.1 The complementarity formulation

Substituting the definition of the ex-ante real rate into the rule gives a fixed-point condition in i_t :

$$i_t = \max\{-\mu, i_t - a_t\}, \quad a_t \equiv \mathbb{E}_t\pi_{t+1} - \phi\pi_t. \quad (\text{D.1})$$

Consider the two branches of the max operator:

- *Unconstrained (P)*: the second argument is selected, so $i_t = i_t - a_t \iff a_t = 0$, and selection requires $i_t - a_t \geq -\mu \iff i_t \geq -\mu$.
- *Constrained (Z)*: the first argument is selected, so $i_t = -\mu$, and selection requires $-\mu \geq i_t - a_t = -\mu - a_t \iff a_t \geq 0$.

Defining $h_t \equiv i_t + \mu$, both branches – and no other configurations – are equivalent to the single equation

$$\min\{a_t, h_t\} = 0, \quad (\text{D.2})$$

since $\min\{a, h\} = 0$ holds iff $(a = 0 \text{ and } h \geq 0)$ or $(h = 0 \text{ and } a \geq 0)$. The nominal rate is backed out of the DISE,

$$i_t = \widehat{\sigma}^{-1}(\delta\mathbb{E}_t x_{t+1} - x_t + \varepsilon_t) + \mathbb{E}_t\pi_{t+1}. \quad (\text{D.3})$$

Note that the Z-branch consistency condition is $a_t \geq 0 \iff \mathbb{E}_t\pi_{t+1} \geq \phi\pi_t$: it restricts the *sign* of expected disinflation and does not involve μ . Under a Taylor rule the corresponding condition is $\phi\pi_t \leq -\mu$, a *level* condition requiring deflation of order μ/ϕ .

Stacking the two Markov states with $\mathbf{y} = (x_1, \pi_1, x_2, \pi_2)^\top$, the MSV solutions are exactly the zeros of the map $\mathbf{G}(\mathbf{y})$ whose rows are, for each state $j \in \{1, 2\}$,

$$g_j(\mathbf{y}) = \min \{a_j(\mathbf{y}), h_j(\mathbf{y})\}, \quad n_j(\mathbf{y}) = \pi_j - \beta(\mathbf{K}\boldsymbol{\pi})_j - \kappa x_j,$$

with $a_j = (\mathbf{K}\boldsymbol{\pi})_j - \phi\pi_j$ and $h_j = -(\kappa\widehat{\sigma})^{-1}\kappa [x_j - \delta(\mathbf{K}\mathbf{x})_j - \varepsilon_j] + (\mathbf{K}\boldsymbol{\pi})_j + \mu$. Since the minimum of two affine functions is continuous and piecewise-linear, $\mathbf{G}$ is a continuous piecewise-linear map, and the GLM theorem applies to it. Its four linear pieces correspond to the regime configurations $s \in \{PP, ZP, PZ, ZZ\}$, where each state's g -row is the gradient of a_j (P) or of h_j (Z).

D.2 Piece Jacobians and the Schur reduction

The gradients of the three row types with respect to $(\mathbf{x}, \boldsymbol{\pi})$ are

$$\begin{aligned} \nabla a_j &= \left(\mathbf{0}, \text{row}_j(\mathbf{K} - \phi\mathbf{I}) \right), \\ \nabla h_j &= \left(-\widehat{\sigma}^{-1} \text{row}_j(\mathbf{I} - \delta\mathbf{K}), \text{row}_j(\mathbf{K}) \right), \\ \nabla n_j &= \left(-\kappa \mathbf{e}_j^\top, \text{row}_j(\mathbf{I} - \beta\mathbf{K}) \right). \end{aligned}$$

Order the rows as (g_1, g_2, n_1, n_2) and the columns as (x_1, x_2, π_1, π_2) ; relative to the natural per-state ordering this swaps one row pair and one column pair, so the permutation signs cancel and determinants are unaffected. The piece Jacobian then has the block form

$$J_s = \begin{pmatrix} \mathbf{G}_s^x & \mathbf{G}_s^\pi \\ -\kappa\mathbf{I} & \mathbf{I} - \beta\mathbf{K} \end{pmatrix},$$

and since $-\kappa\mathbf{I}$ is invertible, the Schur complement formula gives

$$\det J_s = \det(-\kappa\mathbf{I}) \cdot \det \mathbf{R}_s = \kappa^2 \det \mathbf{R}_s, \quad \mathbf{R}_s \equiv \mathbf{G}_s^\pi + \kappa^{-1} \mathbf{G}_s^x (\mathbf{I} - \beta\mathbf{K}), \quad (\text{D.4})$$

where the block-swap sign $(-1)^n$ of the Schur step equals $+1$ for $n = 2$. For a P-row, $\mathbf{G}^x$ vanishes and the contribution to $\mathbf{R}_s$ is $\text{row}_j(\mathbf{K} - \phi\mathbf{I})$. For a Z-row,

$$\text{row}_j(\mathbf{K}) - (\kappa\widehat{\sigma})^{-1} \text{row}_j[(\mathbf{I} - \delta\mathbf{K})(\mathbf{I} - \beta\mathbf{K})] = -(\kappa\widehat{\sigma})^{-1} \text{row}_j[(\mathbf{I} - \delta\mathbf{K})(\mathbf{I} - \beta\mathbf{K}) - \kappa\widehat{\sigma}\mathbf{K}] = -(\kappa\widehat{\sigma})^{-1} \text{row}_j(\mathbf{Q}),$$

with $\mathbf{Q}$ exactly as in Section A (entries $Q_{11}, Q_{22}, Q_{12} = -(1-p)\gamma, Q_{21} = -(1-q)\gamma$).

D.3 The four piece determinants

PP. Both rows come from $\mathbf{K} - \phi\mathbf{I}$:

$$\det J_{PP} = \kappa^2 \det(\mathbf{K} - \phi\mathbf{I}) = \kappa^2 [(p - \phi)(q - \phi) - (1 - p)(1 - q)] = \kappa^2(\phi - 1)(\phi - \rho), \quad (\text{D.5})$$

using $\phi^2 - (p + q)\phi + (p + q - 1) = (\phi - 1)(\phi - \rho)$ with $\rho \equiv p + q - 1$.

ZZ. Both rows come from $-(\kappa\widehat{\sigma})^{-1}\mathbf{Q}$:

$$\det J_{ZZ} = \kappa^2 (\kappa\widehat{\sigma})^{-2} \det \mathbf{Q} = \frac{\det \mathbf{Q}}{\widehat{\sigma}^2}. \quad (\text{D.6})$$

As required, this is proportional to the Taylor-rule ZZ determinant $\det \mathbf{M}_{ZZ} \cdot D_\beta = \det \mathbf{Q}$: the fully constrained regime is rule-independent.

ZP (state 1 at the ELB, state 2 unconstrained).

$$\begin{aligned} \det J_{ZP} &= \kappa^2 \det \begin{pmatrix} -(\kappa\widehat{\sigma})^{-1}Q_{11} & -(\kappa\widehat{\sigma})^{-1}Q_{12} \\ 1 - q & q - \phi \end{pmatrix} = \frac{\kappa}{\widehat{\sigma}} [(\phi - q)Q_{11} + (1 - q)Q_{12}] \\ &= \frac{\kappa}{\widehat{\sigma}} [(\phi - q)Q_{11} - (1 - p)(1 - q)\gamma]. \end{aligned} \quad (\text{D.7})$$

PZ (state 1 unconstrained, state 2 at the ELB). By the symmetric computation,

$$\det J_{PZ} = \frac{\kappa}{\widehat{\sigma}} [(\phi - p)Q_{22} + (1 - p)Q_{21}] = \frac{\kappa}{\widehat{\sigma}} [(\phi - p)Q_{22} - (1 - p)(1 - q)\gamma]. \quad (\text{D.8})$$

D.4 Simplification for $q = 1$ and the sign pattern

At $q = 1$: $\rho = p$, $Q_{11} = (1 - \delta p)(1 - \beta p) - \kappa\widehat{\sigma}p$, $Q_{22} = (1 - \delta)(1 - \beta) - \kappa\widehat{\sigma} = H$, $Q_{21} = 0$, and $\det \mathbf{Q} = Q_{11}H$. Hence

$$\det J_{PP} = \kappa^2(\phi - 1)(\phi - p), \quad \det J_{ZP} = \frac{\kappa(\phi - 1)}{\widehat{\sigma}} Q_{11}, \quad \det J_{PZ} = \frac{\kappa(\phi - p)}{\widehat{\sigma}} H, \quad \det J_{ZZ} = \frac{Q_{11}H}{\widehat{\sigma}^2}. \quad (\text{D.9})$$

With an active rule ($\phi > 1$): under IADL ($\widehat{\sigma} < 0, 0 < \delta < 1$), $H > 0$ and $Q_{11} > 0$, so the signs are $\{+, -, -, +\}$; under standard demand ($\widehat{\sigma} > 0$) with $H < 0$ (which holds at

every calibration in the main text), the signs are $\{+, \text{sgn } Q_{11}, -, -\text{sgn } Q_{11}\}$, both vectors in the order PP, ZP, PZ, ZZ of (D.9). The same-sign condition fails in both of these cases, so the GLM uniqueness property cannot hold under an active RR under IADL or under standard demand with $H < 0$; for $\widehat{\sigma} > 0$ with $H > 0$ all four determinants are positive and uniqueness obtains.

Why a stacked regime-by-regime construction is invalid here. Suppose one encodes the unconstrained regime by its solved restrictions ($\pi_t = \frac{\kappa}{1-\beta\phi}x_t$ and $\pi_t = \phi^{-1}\mathbb{E}_t\pi_{t+1}$) as rows of regime matrices, and applies the same-sign test to those determinants. Under the Taylor rule this is legitimate because all four regimes share the single reduced IS equation, so the stacked matrices coincide with the piece Jacobians of a continuous map. Under the RR they do not: the unconstrained regime imposes a different *equation*, the pieces fail to agree on the regime boundaries, and the resulting determinants differ from (D.5)–(D.8) by regime-*dependent* factors. Those factors flip the sign of the mixed-regime determinants precisely in the IADL region, so the stacked construction would incorrectly pass the same-sign test there. The continuity of the piecewise-linear map is a precondition of the GLM theorem, and only the complementarity formulation above satisfies it.

D.5 Direct enumeration at $q = 1$

Since the same-sign condition fails, uniqueness must be settled by direct enumeration. At $q = 1$ the transition matrix is $K = \begin{pmatrix} p & 1-p \\ 0 & 1 \end{pmatrix}$, state 2 is absorbing with $\varepsilon_2 = 0$, and state 1 carries the shock ε^T . Each state is in branch P ($a_j = 0, h_j \geq 0$) or branch Z ($h_j = 0, a_j \geq 0$). We solve all four configurations exactly. Throughout, Q_{11} and H denote their $q = 1$ values above.

(P,P). State 2: $a_2 = \pi_2 - \phi\pi_2 = 0 \Rightarrow \pi_2 = 0$ (for $\phi \neq 1$), and the NKPC gives $x_2 = 0$; the DISE then yields $i_2 = 0 \geq -\mu$. State 1: $a_1 = p\pi_1 - \phi\pi_1 = 0 \Rightarrow \pi_1 = 0$ (for $\phi \neq p$), $x_1 = 0$ by the NKPC, and the DISE gives $i_1 = \varepsilon^T/\widehat{\sigma}$. This is the *insulated* solution: inflation and the output gap are closed for any shock, with the real rate absorbing it. Validity: $h_1 \geq 0 \iff \varepsilon^T/\widehat{\sigma} \geq -\mu$; under IADL ($\widehat{\sigma} = -|\widehat{\sigma}|$), this is $\varepsilon^T \leq \bar{\varepsilon}_1 \equiv |\widehat{\sigma}|\mu$.

(Z,P). State 2 as above: $\pi_2 = x_2 = 0$. State 1 at the ELB: the DISE and NKPC give

$$x_1 = \delta p x_1 + \widehat{\sigma}(\mu + p\pi_1) + \varepsilon^T, \quad (1 - \beta p)\pi_1 = \kappa x_1.$$

Substituting the second into the first and collecting terms,

$$\pi_1 [(1 - \delta p)(1 - \beta p) - \kappa \widehat{\sigma} p] = \kappa(\widehat{\sigma} \mu + \varepsilon^T) \implies \pi_1 = \frac{\kappa(\widehat{\sigma} \mu + \varepsilon^T)}{Q_{11}}. \quad (\text{D.10})$$

Validity: $a_1 \geq 0 \iff p\pi_1 \geq \phi\pi_1 \iff (\phi - p)\pi_1 \leq 0$. Under IADL $Q_{11} > 0$, so for an active rule ($\phi > 1 > p$) validity is $\pi_1 \leq 0 \iff \varepsilon^T \leq \bar{\varepsilon}_1$ – the *same* half-line on which (P,P) is valid, not its complement. At $\varepsilon^T = 0$ the validity condition reads $Q_{11}/(\kappa\widehat{\sigma}) \leq 0$, which is the existence condition for absorbing ELB equilibria derived by [Holden \(2024, Supplemental Appendix K.8\)](#), with his ELB-state persistence mapped to our p ; under IADL it holds identically in the parameters.

(P,Z). State 2 at the ELB is an absorbing liquidity trap:

$$x_2(1 - \delta) = \widehat{\sigma}(\mu + \pi_2), \quad (1 - \beta)\pi_2 = \kappa x_2 \implies \pi_2 = \frac{\kappa \widehat{\sigma} \mu}{(1 - \delta)(1 - \beta) - \kappa \widehat{\sigma}} = \frac{\kappa \widehat{\sigma} \mu}{H}. \quad (\text{D.11})$$

Validity in state 2: $a_2 = \pi_2 - \phi\pi_2 \geq 0 \iff (\phi - 1)\pi_2 \leq 0$. Under IADL $H > 0$, so $\pi_2 < 0$ and the trap is valid for *any* $\phi > 1$ and any $\mu > 0$. State 1 is unconstrained: $a_1 = 0$ gives $p\pi_1 + (1 - p)\pi_2 = \phi\pi_1$, hence

$$\pi_1 = \frac{(1 - p)\pi_2}{\phi - p}, \quad x_1 = \frac{(1 - \beta\phi)\pi_1}{\kappa},$$

where the expression for x_1 follows from the NKPC after substituting $\mathbb{E}_1 \pi' = \phi\pi_1$. The state-1 nominal rate from the DISE is $i_1 = \phi\pi_1 + \widehat{\sigma}^{-1} [\delta (px_1 + (1 - p)x_2) - x_1 + \varepsilon^T]$, and validity requires $h_1 = i_1 + \mu \geq 0$. Under IADL, solving $i_1 \geq -\mu$ for ε^T (the inequality flips when multiplying by $\widehat{\sigma} < 0$) gives $\varepsilon^T \leq \bar{\varepsilon}_2$ with

$$\bar{\varepsilon}_2 = (1 - \delta p)x_1 - \delta(1 - p)x_2 - \widehat{\sigma}(\mu + \phi\pi_1) = |\widehat{\sigma}|\mu + \frac{(1 - p)\pi_2}{\kappa(\phi - p)} [(1 - \delta p)(1 - \beta\phi) - \delta(1 - \beta)(\phi - p) - \kappa\widehat{\sigma}\phi], \quad (\text{D.12})$$

where the second equality substitutes x_1 , x_2 , and π_1 in terms of π_2 and uses $x_2 = (1 - \beta)\pi_2/\kappa$.

(Z,Z). State 2 as in (P,Z). State 1 at the ELB solves

$$x_1 = \delta [px_1 + (1 - p)x_2] + \widehat{\sigma} [\mu + p\pi_1 + (1 - p)\pi_2] + \varepsilon^T, \quad \pi_1 = \beta [p\pi_1 + (1 - p)\pi_2] + \kappa x_1,$$

with validity $a_1 \geq 0 \iff p\pi_1 + (1-p)\pi_2 \geq \phi\pi_1$. At the shock value $\varepsilon^T = \bar{\varepsilon}_2$ at which state 1 sits exactly at the kink of the rule – $i_1 = -\mu$ and $\phi\pi_1 = \mathbb{E}_1\pi'$ simultaneously – the (P,Z) and (Z,Z) solutions coincide. Under IADL with $\phi > p$ both validity conditions are decreasing in ε^T : h_1 has slope $1/\widehat{\sigma} < 0$ in (P,Z), and a_1 has slope $(p-\phi)\kappa/Q_{11} < 0$ in (Z,Z), since $Q_{11} > 0$. Hence the two configurations are valid on the same half-line $\varepsilon^T \leq \bar{\varepsilon}_2$, with the boundary attained at the shared kink solution, and the two cutoffs are identical by construction rather than merely close.

Counting. Under IADL with $\phi > 1$: all four configurations are valid for $\varepsilon^T < \min\{\bar{\varepsilon}_1, \bar{\varepsilon}_2\}$; exactly one pair – (P,P)/(Z,P) or (P,Z)/(Z,Z) – survives for ε^T between the thresholds; and none is valid for $\varepsilon^T > \max\{\bar{\varepsilon}_1, \bar{\varepsilon}_2\}$. Since $\bar{\varepsilon}_1 = |\widehat{\sigma}|\mu > 0$, the multiplicity region contains $\varepsilon^T = 0$ and all $\varepsilon^T < 0$: a self-fulfilling liquidity trap coexists with the insulated equilibrium even absent any shock. Conversely, uniqueness for every ε^T requires $\phi < 1$ (to invalidate the trap, since $(\phi-1)\pi_2 \leq 0$ with $\pi_2 < 0$ fails only then) and $\phi < p$ (so that the (Z,P) validity $(\phi-p)\pi_1 \leq 0$ selects $\pi_1 \geq 0 \iff \varepsilon^T \geq \bar{\varepsilon}_1$, the exact complement of the (P,P) region); as $p < 1$, the binding requirement is $\phi < p$, which violates the Blanchard-Kahn conditions of the unconstrained regime.

Numerical verification. All results in this section are verified in `Julia/rr_enumeration_check.jl` (replication package): the closed forms (D.5)–(D.8) match the literal piece Jacobians of G to a maximum relative error of order 10^{-12} over 14,640 determinants; the enumeration reproduces the counts 4/2/0 with thresholds $\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$ (e.g. 0.0080 and 0.0087 at $\delta = 0.88$, $\widehat{\sigma} = -0.4$, $\phi = 2$, $p = 0.9$, $q = 1$, $\mu = 0.02$); the same machinery applied to the Taylor rule reproduces the results of the main text exactly (a single valid MSV solution for all shock sizes under passive policy with IADL); and a sweep over $(\lambda, \chi, \phi, p, q, \varepsilon^T)$ at the calibrated $\sigma = 2$ ($\lambda \in \{0.05, 0.15, \dots, 0.95\}$, $\chi \in \{0.1, 0.5, \dots, 4.9\}$, $\phi \in \{1.2, 2, 5\}$, $p \in \{0.3, 0.7, 0.9, 1\}$, $q \in \{0.6, 0.9, 1\}$, and five shock values) finds no parameter point at which the RR admits exactly one MSV solution with an active rule.

E Yield-Curve Control Determinants

This section provides the step-by-step derivation of the regime determinants for the HANK model under always-on yield-curve control (Section 3). A fraction $\omega_r \in (0, 1)$ of savers (r-type) hold long government bonds at the YCC-pegged yield $i_t^L = \phi_L \pi_t$; the complemen-

tary fraction holds short government bonds at the Taylor-rule yield $i_t = \max\{-\mu, \phi\pi_t\}$. The aggregate IS (15) is the goods-market clearing condition with type-weighted Euler equations.

E.1 Regime structure

Substituting the policy rules into the aggregate IS (15) and dropping the regime-dependent intercept $+\widehat{\sigma}(1-\omega_r)\mu$ of a state at the ELB (which affects the solution but not the coefficient matrix), the effective coefficient on π entering the IS in state s is

$$\phi_s^{\text{eff}} = \begin{cases} \bar{\phi} \equiv (1-\omega_r)\phi + \omega_r\phi_L & \text{state } s \text{ active,} \\ \omega_r\phi_L & \text{state } s \text{ at the ELB.} \end{cases} \quad (\text{E.1})$$

The diagonal matrix $\mathbf{\Phi}_s = \text{diag}(\phi_{s,1}^{\text{eff}}, \phi_{s,2}^{\text{eff}})$ collects the per-state coefficients.

Stacked-system reduction. Writing the IS in stacked form across the two Markov states:

$$x = \delta Kx - \widehat{\sigma}[\mathbf{\Phi}_s\pi - K\pi] + \varepsilon_s.$$

Substituting $\pi = Cx$ with $C = \kappa(I - \beta K)^{-1}$, then right-multiplying by $(I - \beta K)$ and using $C(I - \beta K) = \kappa I$:

$$M_s^{\text{YCC}}(I - \beta K) = (I - \delta K)(I - \beta K) + \widehat{\sigma}\kappa(\mathbf{\Phi}_s - K) = Q + \widehat{\sigma}\kappa\mathbf{\Phi}_s, \quad (\text{E.2})$$

where $Q \equiv (I - \delta K)(I - \beta K) - \widehat{\sigma}\kappa K$ is the standard Taylor-rule Q -matrix from (H.4). Equation (E.2) is the central structural result: the YCC regime distinction enters as a *diagonal* perturbation of the TR Q -matrix.

E.2 General- (p, q) determinant

Taking determinants in (E.2):

$$\boxed{\det M_s^{\text{YCC}} \cdot D_\beta = \det(Q + \widehat{\sigma}\kappa\mathbf{\Phi}_s).} \quad (\text{E.3})$$

Expanding the 2×2 determinant gives the closed form valid at any (p, q) :

$$\det \mathbf{M}_s^{YCC} \cdot D_\beta = (Q_{11} + \widehat{\sigma}\kappa \phi_{s,1}^{\text{eff}})(Q_{22} + \widehat{\sigma}\kappa \phi_{s,2}^{\text{eff}}) - Q_{12}Q_{21}. \quad (\text{E.4})$$

The four regimes differ only in the assignment of $\bar{\phi}$ vs $\omega_r \phi_L$ to the two diagonal entries of Φ_s .

Contrast with the long-rate rules. Under the long-rate rules (Section F), the consol recursion multiplies the policy coefficient by the amplification factor $(1 - \beta\mu)/(1 - \beta)$, both in the eigenvalue factors $\mathcal{E}_\mu^{nLR}$ and in the mixed-regime coefficients a^{PZ}, a^{ZP} , and a state at the ELB carries no policy term at all. Under YCC the ELB state retains the coefficient $\omega_r \phi_L$. Under YCC the consol is absent at the IS level – the long rate enters as a separate object weighted by ω_r , not via a forward-recursion in the IS – so no amplification arises and the eigenvalue factor reverts to the standard TR form.

E.3 Simplification for $q = 1$

At $q = 1$ the Markov kernel K is upper triangular and Q inherits this structure with $Q_{21}|_{q=1} = 0$. The diagonal entries factor as

$$\begin{aligned} Q_{11}|_{q=1} &= (1 - \delta p)(1 - \beta p) - \widehat{\sigma}\kappa p = \mathcal{E}_p(0), \\ Q_{22}|_{q=1} &= (1 - \delta)(1 - \beta) - \widehat{\sigma}\kappa = \mathcal{E}_1(0), \end{aligned}$$

where the TR-style eigenvalue factor is

$$\mathcal{E}_\mu(\phi) \equiv (1 - \delta\mu)(1 - \beta\mu) + \widehat{\sigma}\kappa(\phi - \mu). \quad (\text{E.5})$$

Equation (E.4) collapses to a product:

$$\boxed{\det \mathbf{M}_s^{YCC} \cdot D_\beta \Big|_{q=1} = \mathcal{E}_p(\phi_{s,1}^{\text{eff}}) \cdot \mathcal{E}_1(\phi_{s,2}^{\text{eff}}).} \quad (\text{E.6})$$

Explicitly for the four regimes, with ϕ^{eff} assigned by (E.1):

$$\det \mathbf{M}_{PP}^{\text{YCC}} \cdot D_\beta|_{q=1} = \mathcal{E}_p(\bar{\phi}) \mathcal{E}_1(\bar{\phi}), \quad \det \mathbf{M}_{PZ}^{\text{YCC}} \cdot D_\beta|_{q=1} = \mathcal{E}_p(\bar{\phi}) \mathcal{E}_1(\omega_r \phi_L), \quad (\text{E.7})$$

$$\det \mathbf{M}_{ZP}^{\text{YCC}} \cdot D_\beta|_{q=1} = \mathcal{E}_p(\omega_r \phi_L) \mathcal{E}_1(\bar{\phi}), \quad \det \mathbf{M}_{ZZ}^{\text{YCC}} \cdot D_\beta|_{q=1} = \mathcal{E}_p(\omega_r \phi_L) \mathcal{E}_1(\omega_r \phi_L). \quad (\text{E.8})$$

Four distinct products of TR eigenvalue factors – non-degenerate, with policy argument equal to the active-state effective coefficient $\bar{\phi}$ or the ELB-state coefficient $\omega_r \phi_L$.

E.4 Four-sign condition and policy threshold

The GLM uniqueness condition requires all four products in (E.7)–(E.8) to share the same sign, which (because each $\mathcal{E}_\mu$ takes only two values across the four regimes: $\mathcal{E}_\mu(\bar{\phi})$ or $\mathcal{E}_\mu(\omega_r \phi_L)$) reduces to

$$\text{sgn } \mathcal{E}_\mu(\bar{\phi}) = \text{sgn } \mathcal{E}_\mu(\omega_r \phi_L) \quad \mu \in \{1, p\}. \quad (\text{E.9})$$

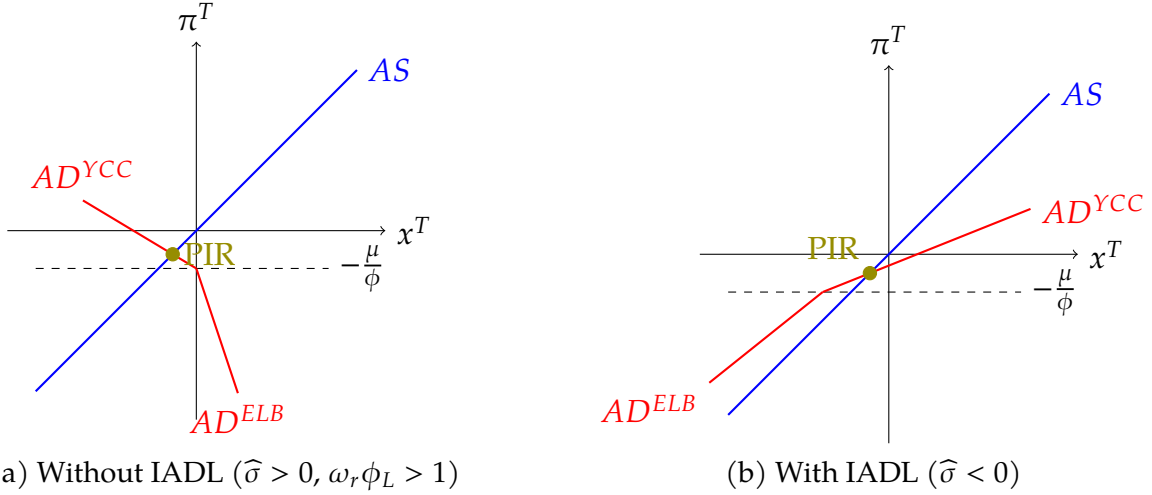
Each $\mathcal{E}_\mu$ is linear in ϕ with slope $\widehat{\sigma}\kappa$; its zero is at $\phi_\mu^* = \mu - (1 - \delta\mu)(1 - \beta\mu)/(\widehat{\sigma}\kappa)$.

Non-IADL case ($\widehat{\sigma} > 0$). Each $\mathcal{E}_\mu$ is increasing in ϕ . The intercept $\mathcal{E}_\mu(0) = (1 - \delta\mu)(1 - \beta\mu) - \widehat{\sigma}\kappa\mu$ is negative for plausible HANK calibrations because $(1 - \delta\mu)(1 - \beta\mu)$, at most of order 10^{-2} at the calibrations of Table 1 with $p = 0.9$, is dominated by $\widehat{\sigma}\kappa\mu \sim 10^{-1}$. The same-sign condition is satisfied with both factors strictly positive iff $\min(\bar{\phi}, \omega_r \phi_L) > \max(\phi_1^*, \phi_p^*)$; when, as at the calibrations of Table 1, $\phi_p^* < \phi_1^*$, every other sign-consistent configuration places $\bar{\phi}$ below $\phi_1^* = \phi_{\text{low}}^*$ and so violates the Blanchard-Kahn condition (20). Since $\omega_r \phi_L \leq \bar{\phi}$, the binding constraint is $\omega_r \phi_L > \max(\phi_1^*, \phi_p^*)$; at the calibrations of Table 1 (with $q = 1$ and p close to one) the max is ϕ_1^* , so

$$\boxed{\omega_r \phi_L > \phi_1^* \approx 1.} \quad (\text{E.10})$$

This is the YCC analogue of the Taylor principle, applied to the effective inflation response *at the ELB regime*. The correction term $-(1 - \delta)(1 - \beta)/(\widehat{\sigma}\kappa)$ is a few times 10^{-3} in absolute value, so $\omega_r \phi_L > 1$ is an accurate approximation for most calibrations; the exact threshold is $\phi_1^* = 1 + (\delta - 1)(1 - \beta)/(\widehat{\sigma}\kappa)$, which exceeds 1 when $\delta > 1$ (1.0049 at the GKN calibration).

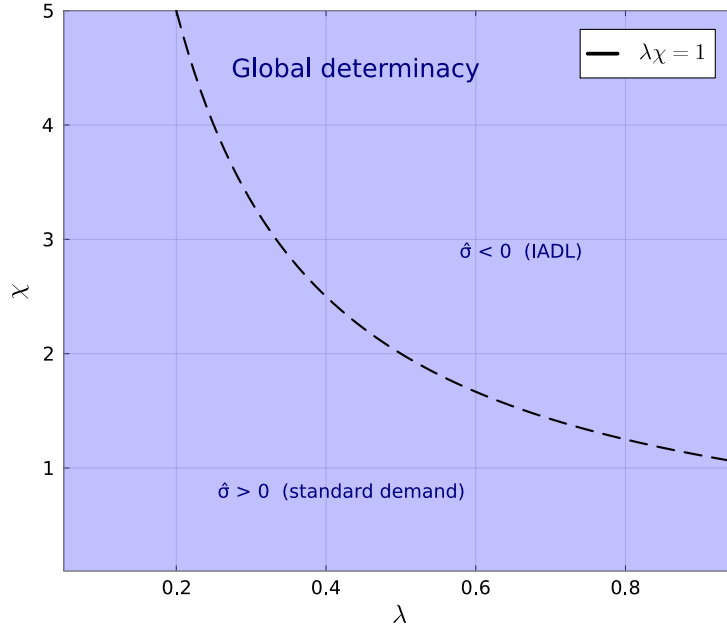
Figure E.1: AD-AS Schedules in the HANK Model under Yield-Curve Control (Transitory State)



Note: Figure shows AS (blue) and the kinked piecewise-linear AD (red) under always-on YCC in the transitory state, at $q = 1$ conditioning on the absorbing state at the intended steady state, for a small contractionary shock. The kink sits on the short-rate floor line $\pi^T = -\mu/\phi$, where the Taylor-rule short rate reaches $-\mu$ while the pegged long-rate channel stays operative; AD slopes are $-(1 - \delta p)/[\widehat{\sigma}(\phi^{\text{eff}} - p)]$ with $\phi^{\text{eff}} = \bar{\phi}$ on the active segment and $\omega_r \phi_L$ on the ELB segment, the latter the steeper since $\omega_r \phi_L < \bar{\phi}$. Left panel (non-IADL, $1 - \delta p > 0$, $\omega_r \phi_L > 1$): both segments slope down, so the kinked curve crosses the upward AS exactly once. Right panel (IADL with both effective coefficients above p , uniqueness branch (ii) of §H.3): both segments slope up but stay flatter than AS, again a single crossing. The schedules are schematic, drawn to the slope signs and ordering at $\beta = 0.99$, $\kappa = 0.1769$, $\phi = 1.5$, $\omega_r = 0.5$, $\phi_L = 2.5$ (so $\omega_r \phi_L = 1.25$), and $p = 0.9$, with $(\delta, \widehat{\sigma}) = (1.10, 0.875)$ in the left panel and $(0.84, -0.5)$ in the right; the slopes, the kink, and the single crossing at this calibration are verified in Julia/verify_AD_YCC.jl. The two-shock comparative statics for the non-IADL case are in Figure 4 of the main text.

IADL case ($\widehat{\sigma} < 0$, $\delta < 1$). Each $\mathcal{E}_\mu$ is decreasing in ϕ with $\mathcal{E}_\mu(0) > 0$. Two regions pass the four-sign condition: (i) both factors positive when $\max(\bar{\phi}, \omega_r \phi_L) = \bar{\phi} < \min(\phi_1^*, \phi_p^*)$, i.e. the policy response is small enough to stay below the eigenvalue zero; (ii) both factors negative when $\min(\bar{\phi}, \omega_r \phi_L) = \omega_r \phi_L > \max(\phi_1^*, \phi_p^*)$. Between these two regions lies a forbidden zone where the four-sign condition fails. Once local determinacy is imposed the two branches are not symmetric. Under IADL the trace-root condition of §B.6 is $\bar{\phi} < \phi_1^*$, the IADL Taylor-principle bound of the main text; it is necessary for the Blanchard-Kahn conditions on the first of the two admissible intervals of $\bar{\phi}$ identified there, and branch (i) satisfies it. Branch (ii) has $\bar{\phi} \geq \omega_r \phi_L > \phi_1^*$, so it violates local determinacy unless $\bar{\phi}$ reaches the second, very aggressive interval, on which $D < 0$. That interval begins at $\bar{\phi} \approx 40$ at the illustrative calibration $(\delta, \widehat{\sigma}) = (0.84, -0.5)$ of Figure E.1, and comes within reach of the search behind Figure E.2 only where $|\widehat{\sigma}|$ is large, just above the line $\lambda_\chi = 1$.

Figure E.2: Global Determinacy Region for the HANK Model under Yield-Curve Control



Note: Numerical search with $\beta = 0.99$, $\sigma = 2$, $\kappa = 0.1769$, $\phi = 1.5$, $s = 0.96$, $\omega_r = 0.5$, $q = 1$, and $p \in [0.85, 0.95]$. For each (λ, χ) we search the long-rate intensity ϕ_L over 401 points of $[0, 20]$: a point is shaded when some $\omega_r \phi_L$ delivers both a unique MSV solution and the BK condition at this scope, so the whole plane is a global-determinacy region. The dashed line is the $\hat{\sigma}$ -sign boundary $\lambda\chi = 1$. *Below* it (standard aggregate demand, $\hat{\sigma} > 0$) the binding condition is the YCC analogue of the Taylor principle, $\omega_r \phi_L > 1$ (Proposition 3) – precisely the region that is locally indeterminate under the Taylor rule (Figure 1) and lacks a unique MSV solution under the RR (Proposition 2), here rescued by the long-rate channel that stays active at the ELB. *Above* it (IADL, $\hat{\sigma} < 0$) global determinacy is obtained, at most points through the opposite configuration: the admissible *effective* response $\tilde{\phi} = (1 - \omega_r)\phi + \omega_r \phi_L$ is passive, below the eigenvalue zero $\phi_1^* \approx 1$, consistent with the passive-policy requirement of IADL under a Taylor rule (Billbie, 2008). The active branch satisfying the uniqueness condition violates local determinacy except at very aggressive responses, which the search reaches only in a band just above $\lambda\chi = 1$ where $|\hat{\sigma}|$ is large (§E.4).

At plausible policy responses the globally determinate IADL configuration is therefore branch (i).

Numerical verification. The general- (p, q) formula (E.3) and the $q = 1$ factorisation (E.7)–(E.8) are verified against direct construction of M_s^{YCC} from (15) in Julia/verify_YCC_determinants.jl across 38 400 parameter combinations spanning $(\lambda, \chi) \in [0.05, 0.95] \times [0.1, 5.0]$, four Markov (p, q) pairs, and the YCC parameters (ω_r, ϕ, ϕ_L) . Maximum relative error is below 10^{-10} at general (p, q) and $\sim 10^{-12}$ on the $q = 1$ subset where the factorisation (E.6) is independently checked. The companion script Julia/verify_AD_YCC.jl verifies

the AD schedules used in Figure E.1 and in Figure 4 of the main text: the slopes against finite differences of the IS equation, the kink against the short-rate floor line $\pi^T = -\mu/\phi$, and, at the non-IADL calibrations, the uniqueness of the regime-consistent configuration at every shock.

Figure E.2 applies the conditions of this subsection pointwise: at each (λ, χ) on the verified grid, we look for *any* searched $\omega_r \phi_L$ at which the four-sign condition (E.9) and the Blanchard-Kahn condition both hold. Because ϕ_L is chosen pointwise and such a value exists at every grid point, the shaded region covers the whole plane, with no interior boundary to trace. The main text (§3) therefore states this result in prose rather than in a figure of its own.

F Long-Term Nominal Interest Rate Rule Determinants

This section records the determinant algebra for the long-term nominal interest rate rule (nLRR), the unsegmented alternative to the segmented YCC of main-text Section 3. The nLRR is $i_t^L = \phi_\pi \pi_t$, where the nominal consol satisfies the expectations-hypothesis recursion $i_t^L = (1 - \beta) i_t + \beta \mathbb{E}_t i_{t+1}^L$. As established in §H.4, the nLRR fails global determinacy under non-IADL because the eigenvalue intercepts $\mathcal{E}_\mu^{nLRR}(0)$ are dominated by $\widehat{\sigma} \kappa \mu$ at $\beta \approx 1$, forcing BK and GLM into mutually exclusive intervals. The detailed algebra is recorded here for reference; the paper's working specification is the segmented YCC of Section E.

F.1 Regime structure under the nLRR

Because i_t appears explicitly in the nominal consol recursion, the ELB $i_t \geq -\mu$ binds on a variable that lives inside the solved system. Combining the consol recursion with the rule and applying the NKPC identity $\pi_t - \beta \mathbb{E}_t \pi_{t+1} = \kappa x_t$ gives the short nominal rate

$$i_t = \frac{\phi_\pi}{1 - \beta} [\pi_t - \beta \mathbb{E}_t \pi_{t+1}] = \frac{\phi_\pi \kappa}{1 - \beta} x_t. \quad (\text{F.1})$$

Fisher's identity then leaves $\mathbb{E}_t \pi_{t+1}$ in the DISE (in contrast to the real LRR of Section G):

$$(\omega I - \delta K)x - \widehat{\sigma} K \pi = \varepsilon, \quad \omega \equiv 1 + \frac{\widehat{\sigma} \kappa \phi_\pi}{1 - \beta}. \quad (\text{F.2})$$

Substituting $\pi = Cx$, $C = \kappa(I - \beta K)^{-1}$, and right-multiplying by $(I - \beta K)$:

$$M_{PP}^{nLRR}(I - \beta K) = (\omega I - \delta K)(I - \beta K) - \widehat{\sigma}\kappa K. \quad (F.3)$$

Validity of the stacked construction. Unlike the robust real rate rule (cf. §D), stacking is legitimate here: all four regimes share the same DISE, NKPC, and consol rows and differ only in which branch of the policy row $\min\{i_j^L - \phi_\pi \pi_j, i_j + \mu\} = 0$ each state selects, so they are the four pieces of one continuous piecewise-linear map, as the GLM theorem requires. That map's piece determinant in regime s equals $\det M_s^{nLRR} \cdot D_\beta$ times the factor left by eliminating i and i^L , namely $\{(1 - \beta)^2, (1 - \beta)(1 - \beta q), (1 - \beta)(1 - \beta p), D_\beta\}$ for $s = \{PP, PZ, ZP, ZZ\}$. Those factors are regime-dependent, as under the RR, but strictly positive for $\beta, p, q \in (0, 1)$, so the same-sign test is unaffected.

Constrained regime. When both states are at the ELB, the system is identical to the Taylor rule case; i^L adjusts endogenously but does not feed back. Hence $\det M_{ZZ}^{nLRR} \cdot D_\beta = \det Q$.

PP determinant via eigenvalue factorisation. The right-hand side of (F.3) is a polynomial in K , whose eigenvalues at the two eigenvalues $\mu \in \{1, \rho\}$ of K (with $\rho \equiv p + q - 1$) are

$$\mathcal{E}_\mu^{nLRR}(\phi) \equiv (1 - \delta\mu)(1 - \beta\mu) + \widehat{\sigma}\kappa \left[\frac{\phi(1 - \beta\mu)}{1 - \beta} - \mu \right], \quad (F.4)$$

so that

$$\boxed{\det M_{PP}^{nLRR} \cdot D_\beta = \mathcal{E}_1^{nLRR}(\phi_\pi) \cdot \mathcal{E}_\rho^{nLRR}(\phi_\pi).} \quad (F.5)$$

Explicitly, $\mathcal{E}_1^{nLRR}(\phi) = (1 - \delta)(1 - \beta) + \widehat{\sigma}\kappa(\phi - 1)$ is linear in ϕ with slope $\widehat{\sigma}\kappa$; $\mathcal{E}_\rho^{nLRR}(\phi)$ is linear with slope $\widehat{\sigma}\kappa(1 - \beta\rho)/(1 - \beta)$, carrying the consol's amplification at the non-unit eigenvalue.

F.2 The nominal consol recursion in mixed regimes

In the mixed regimes, the nominal consol recursion couples the two Markov states: the long nominal rate in the ELB state is endogenous and couples to $\phi_\pi \pi$ in the unconstrained state.

PZ consol recursion (state 1 = nLRR, state 2 = ELB). With $i_1^L = \phi_\pi \pi_1$ and $i_2 = -\mu$, the stacked consol rows are

$$\text{Row 1: } \phi_\pi \pi_1 = (1 - \beta)i_1 + \beta[p \phi_\pi \pi_1 + (1 - p)i_2^L], \quad (\text{F.6})$$

$$\text{Row 2: } i_2^L = -(1 - \beta)\mu + \beta[(1 - q)\phi_\pi \pi_1 + q i_2^L]. \quad (\text{F.7})$$

Step 1: Solve for i_2^L . From (F.7):

$$i_2^L = \frac{-(1 - \beta)\mu + \beta(1 - q)\phi_\pi \pi_1}{1 - \beta q}. \quad (\text{F.8})$$

Step 2: Solve for i_1 . Substituting (F.8) into (F.6) and using the identity $(1 - \beta p)(1 - \beta q) - \beta^2(1 - p)(1 - q) = (1 - \beta)(1 - \beta \rho)$:

$$i_1 = \frac{\phi_\pi(1 - \beta \rho)\pi_1 + \beta(1 - p)\mu}{1 - \beta q}. \quad (\text{F.9})$$

Crucially, i_1 depends on π_1 only (not on π_2); the cross-state channel is entirely collapsed by the nominal consol's algebraic structure. The real LRR of Section G delivers a more complex coupling because its consol recursion contains the real rate ($i_t - \mathbb{E}_t \pi_{t+1}$), which pulls π_2 in through Fisher's identity in the ELB state.

F.3 PZ determinant

Rank-one update. Row 2 of M_{PZ}^{nLRR} coincides with row 2 of M_{ZZ} (state 2 at ELB in both). Row 1 differs from row 1 of M_{ZZ} by the π_1 -coefficient $\widehat{\sigma} a_1^{PZ}$, where (from (F.9) plus Fisher):

$$a_1^{PZ} \equiv \frac{\phi_\pi(1 - \beta \rho)}{1 - \beta q}. \quad (\text{F.10})$$

Hence $M_{PZ}^{nLRR} = M_{ZZ} + \widehat{\sigma} a_1^{PZ} e_1 e_1^\top C$. Right-multiplying by $(I - \beta K)$ and using $C(I - \beta K) = \kappa I$:

$$M_{PZ}^{nLRR}(I - \beta K) = Q + \widehat{\sigma} \kappa a_1^{PZ} e_1 e_1^\top. \quad (\text{F.11})$$

Matrix determinant lemma. For the rank-one update of a 2×2 matrix by $c e_1 e_1^\top$, $\det(A + c e_1 e_1^\top) = \det A + c A_{22}$:

$$\det \mathbf{M}_{PZ}^{nLRR} \cdot D_\beta = \det \mathbf{Q} + \widehat{\sigma} \kappa a_1^{PZ} Q_{22}. \quad (\text{F.12})$$

F.4 ZP determinant

By symmetry $p \leftrightarrow q$, $e_1 \leftrightarrow e_2$:

$$a_2^{ZP} \equiv \frac{\phi_\pi(1 - \beta\rho)}{1 - \beta p}, \quad (\text{F.13})$$

and

$$\det \mathbf{M}_{ZP}^{nLRR} \cdot D_\beta = \det \mathbf{Q} + \widehat{\sigma} \kappa a_2^{ZP} Q_{11}. \quad (\text{F.14})$$

F.5 Simplification for $q = 1$

At $q = 1$: $\rho = p$, K and Q are upper-triangular with $Q_{21}|_{q=1} = 0$. The Q entries factor as

$$Q_{11}|_{q=1} = \mathcal{E}_p^{nLRR}(0) = (1 - \delta p)(1 - \beta p) - \widehat{\sigma} \kappa p, \quad Q_{22}|_{q=1} = \mathcal{E}_1^{nLRR}(0) = (1 - \delta)(1 - \beta) - \widehat{\sigma} \kappa,$$

and the coupling coefficients simplify to $a_1^{PZ}|_{q=1} = \phi_\pi(1 - \beta p)/(1 - \beta)$ and $a_2^{ZP}|_{q=1} = \phi_\pi$.

Four-regime eigenvalue factorisation. All four regime determinants split into a product of one $\mathcal{E}_1^{nLRR}$ -factor and one $\mathcal{E}_p^{nLRR}$ -factor, with policy argument ϕ_π on the unconstrained state and 0 on the ELB state:

$$\begin{aligned} \det \mathbf{M}_{PP}^{nLRR} \cdot D_\beta|_{q=1} &= \mathcal{E}_1^{nLRR}(\phi_\pi) \cdot \mathcal{E}_p^{nLRR}(\phi_\pi), \\ \det \mathbf{M}_{PZ}^{nLRR} \cdot D_\beta|_{q=1} &= \mathcal{E}_1^{nLRR}(0) \cdot \mathcal{E}_p^{nLRR}(\phi_\pi), \end{aligned} \quad (\text{F.15})$$

$$\begin{aligned} \det \mathbf{M}_{ZP}^{nLRR} \cdot D_\beta|_{q=1} &= \mathcal{E}_1^{nLRR}(\phi_\pi) \cdot \mathcal{E}_p^{nLRR}(0), \\ \det \mathbf{M}_{ZZ}^{nLRR} \cdot D_\beta|_{q=1} &= \mathcal{E}_1^{nLRR}(0) \cdot \mathcal{E}_p^{nLRR}(0). \end{aligned} \quad (\text{F.16})$$

Four-sign condition. The MSV solution is unique iff all four determinants share the same sign, which by (F.15)–(F.16) is equivalent to

$$\text{sgn } \mathcal{E}_1^{nLRR}(\phi_\pi) = \text{sgn } \mathcal{E}_1^{nLRR}(0) \quad \text{and} \quad \text{sgn } \mathcal{E}_p^{nLRR}(\phi_\pi) = \text{sgn } \mathcal{E}_p^{nLRR}(0). \quad (\text{F.17})$$

IADL case. Under IADL ($\widehat{\sigma} < 0$, $0 < \delta < 1$), both $\mathcal{E}_1^{nLRR}(0)$ and $\mathcal{E}_p^{nLRR}(0)$ are strictly positive. Each $\mathcal{E}_\mu^{nLRR}$ is linear and strictly decreasing in ϕ , so $\mathcal{E}_\mu^{nLRR}(\phi_\pi) > 0$ iff $\phi_\pi < \phi_\mu^\circ$, where ϕ_μ° is the zero of $\mathcal{E}_\mu^{nLRR}$. The four-sign condition therefore holds under IADL for $\phi_\pi < \min(\phi_1^\circ, \phi_p^\circ)$, an *upper* bound on the policy response, or for $\phi_\pi > \max(\phi_1^\circ, \phi_p^\circ)$. The conflict between the Blanchard-Kahn and uniqueness requirements that rules out the nLRR arises under standard demand, where BK requires $\phi_\pi > \phi_{\text{low}}^*$ (§H.4).

G Pedagogical Note: The Degeneracy of the Real Long-Rate Rule

A natural alternative to the nominal LRR is the *real* long-rate rule $r_t^L = \phi_\pi \pi_t$, where the real consol satisfies $r_t^L = (1 - \beta)(i_t - \mathbb{E}_t \pi_{t+1}) + \beta \mathbb{E}_t r_{t+1}^L$. Algebraically the two specifications coincide at the PP regime's short-rate derivation: combining the rule with the consol recursion and NKPC gives the same $i_t - \mathbb{E}_t \pi_{t+1} = \phi_\pi \kappa x_t / (1 - \beta)$ in both cases. The divergence appears in what happens upon substitution into the DISE.

The cancellation. Under the real rule, the substitution pins down the short *real* rate $r_t = \phi_\pi \kappa x_t / (1 - \beta)$, independent of expected inflation. The DISE becomes

$$\left(1 + \frac{\widehat{\sigma} \kappa \phi_\pi}{1 - \beta}\right) x_t = \delta \mathbb{E}_t x_{t+1} + \varepsilon_t, \quad (\text{G.1})$$

a *one-dimensional* forward equation in x_t alone. Expected inflation has disappeared. The reduced system decouples: π_t becomes a passive variable pinned down by the NKPC once the x_t path is determined.

Why the ELB kink is vacuous. The ELB $i_t \geq -\mu$ is a constraint on a *nominal* variable. In the reduced system (G.1) the nominal rate i_t does not appear: it can be recovered as $i_t = \phi_\pi \kappa x_t / (1 - \beta) + \mathbb{E}_t \pi_{t+1}$, but imposing $i_t \geq -\mu$ does not modify (G.1). Equivalently: the only way the ELB can discipline the reduced dynamics is through a regime switch that kinks x_t 's equation, but the x_t equation contains no term that could kink. Under a direct implementation of the rule (the central bank operates on real long yields via indexed-bond purchases) the short nominal rate is free to breach $-\mu$ without the reduced system noticing. The vacuity is therefore a property of this *direct-implementation* reading rather than of the real long-rate rule as such. If the floor is instead enforced as a constraint on the short

nominal rate, the bank must abandon the real rule when it binds and peg $i_t = -\mu$, so that $r_t = -\mu - \mathbb{E}_t \pi_{t+1}$ and expected inflation re-enters the DISE. The floor regime is then a genuinely two-dimensional pegged-rate system, distinct from the collapsed one-dimensional active regime of (G.1): the four-sign condition is non-vacuous, and it generically fails. Either way the real long-rate rule is not a substantive resolution of the ELB problem.

Consequence for the four-regime analysis. In the real-LRR specification there is only one relevant regime (the PP regime). The GLM four-determinant sign condition used in the Taylor-rule and RR sections collapses to a single determinant, and the requirement that “all four share a sign” is vacuous. Imposing an ELB kink in this setting would be artificial, tacked on rather than generated by the structure of the model.

Resolution. Switching from the real rule $r_t^L = \phi_\pi \pi_t$ to the nominal rule $i_t^L = \phi_\pi \pi_t$ keeps i_t algebraically coupled to π_t through the nominal consol. Expected inflation then survives in the DISE via Fisher’s identity, and the reduced system is genuinely two-dimensional – the ELB kinks i_t , which in turn kinks a variable that lives in the solved system, so the four-regime structure is earned rather than imposed. The nominal rule is therefore the one carried through Section F. The real-rule formulas, obtained by replacing the single PZ coupling a_1^{PZ} with the pair (d_1^{PZ}, d_2^{PZ}) – the second entry being a π_2 coefficient with no nominal-rule counterpart, created by the real consol’s dependence on $\mathbb{E}_t \pi_{t+1}$ in the ELB state – where

$$d_1^{PZ} = \frac{\phi_\pi(1 - \beta\rho) + p(1 - \beta) + \beta(1 - q)}{1 - \beta q}, \quad d_2^{PZ} = \frac{1 - p}{1 - \beta q},$$

(and symmetrically for ZP), are included here purely to document the algebraic point that the β -matched discount in the real consol is the source of the cancellation.

H Existence and Uniqueness of an MSV Solution

This section is the appendix of the main paper, relocated here in full: it carries the proofs of Propositions 1, 2, and 3, which the paper itself now states with short sketches only, together with the analysis of long-rate rules without bond-market segmentation. Equation, table, and figure numbers without a section-letter prefix refer to the main paper.

H.1 Taylor-type rule

We verify existence and uniqueness of an MSV solution for the reference HANK model with a Taylor-type rule using the fixed-point method of [Murakami, Shchapov, and Zhang \(2026\)](#), which builds on the global invertibility conditions of [Gourieroux, Laffont, and Monfort \(1980\)](#) (GLM).

Matrix formulation. Stacking the two Markov states, the system (1)–(3) can be written in matrix form as

$$x = \delta Kx - \widehat{\sigma}(i - K\pi) + \varepsilon, \quad (\text{H.1a})$$

$$\pi = \beta K\pi + \kappa x, \quad (\text{H.1b})$$

$$i = \max\{-\mu\iota, \phi\pi\}, \quad (\text{H.1c})$$

where $x = (x_1, x_2)^\top$, $\pi = (\pi_1, \pi_2)^\top$, $\varepsilon = (\varepsilon_1, \varepsilon_2)^\top$, ι is the vector of ones, and expectations are computed via K . Inverting the NKPC yields

$$\pi = \kappa (I - \beta K)^{-1} x \equiv Cx,$$

where the inverse exists since $\det(I - \beta K) > 0$ for $\beta < 1$. Substituting the NKPC and the Taylor rule into the DISE eliminates inflation and the interest rate, yielding the reduced IS equation

$$Ax + \widehat{\sigma} \max\{-\mu\iota, Bx\} = \varepsilon, \quad (\text{H.2})$$

where

$$A \equiv (I - \delta K) - \widehat{\sigma}KC, \quad B \equiv \phi C. \quad (\text{H.3})$$

Here, A captures the effective IS slope and the expectations channel, while B captures the monetary policy feedback through the Taylor rule.

Gap function and regime matrices. The piecewise-linear mapping (H.2) defines a gap function $G(x) \equiv Ax + \widehat{\sigma} \max\{-\mu\iota, Bx\} - \varepsilon$, whose zeros are the MSV solutions. The ELB constraint creates four regime combinations $s \in \{PP, PZ, ZP, ZZ\}$ (positive or zero interest rate in each Markov state). Under each regime s , the system is linear with coefficient matrix

$$M_s = A + \widehat{\sigma}H_s B,$$

where H_s is a diagonal selection matrix indicating the states in which the policy rate is positive. Explicitly: $H_{PP} = I$, $H_{PZ} = e_1 e_1^\top$, $H_{ZP} = e_2 e_2^\top$, and $H_{ZZ} = 0$.

GLM condition. By the GLM theorem, the piecewise-linear mapping G is a global homeomorphism – so that an MSV solution exists and is unique for every shock vector $\varepsilon \in \mathbb{R}^2$ – if and only if $\det M_s$ share the same sign across all four regimes s . In particular, the same-sign condition is sufficient for a unique MSV solution at every ε^T of the shock family studied in the main text.

Computing the determinants. To evaluate the determinants analytically, we exploit the identity $C(I - \beta K) = \kappa I$ and multiply each M_s by $(I - \beta K)$ on the right. Define

$$Q \equiv (I - \delta K)(I - \beta K) - \kappa \widehat{\sigma} K. \quad (\text{H.4})$$

Then the four regime coefficient matrices, after clearing the inverse $C = \kappa(I - \beta K)^{-1}$, become

$$M_s(I - \beta K) = Q + \kappa \widehat{\sigma} \phi H_s.$$

Since $\det(I - \beta K) > 0$, checking the same-sign condition on $\det M_s$ is equivalent to checking same-sign of $\det[M_s(I - \beta K)]$, i.e.:

$$\begin{aligned} \det M_{PP} \cdot D_\beta &= \det(Q + \kappa \widehat{\sigma} \phi I), & \det M_{PZ} \cdot D_\beta &= \det(Q + \kappa \widehat{\sigma} \phi e_1 e_1^\top), \\ \det M_{ZP} \cdot D_\beta &= \det(Q + \kappa \widehat{\sigma} \phi e_2 e_2^\top), & \det M_{ZZ} \cdot D_\beta &= \det(Q), \end{aligned}$$

where $D_\beta \equiv \det(I - \beta K) > 0$.

Factored determinant expressions. Expanding $\mathbf{Q}$ and computing the 2×2 determinants (detailed step-by-step algebra is provided in §B), we obtain the following factored forms:

$$\det \mathbf{M}_{PP} \cdot D_\beta = \kappa \widehat{\sigma} \left[(1 - \delta)(1 - \beta) + \kappa \widehat{\sigma}(\phi - 1) \right] (\phi - \vartheta), \quad (\text{H.5})$$

$$\begin{aligned} \det \mathbf{M}_{ZP} \cdot D_\beta &= \left[(\kappa \widehat{\sigma})^2(\phi - 1) + \kappa \widehat{\sigma}(1 - \delta)(1 - \beta) \right] (-\vartheta) \\ &\quad - \kappa \widehat{\sigma} \phi(1 - q) \left[\delta(1 - \beta p - \beta q + \beta) + \kappa \widehat{\sigma} + \beta(1 - \delta) \right], \end{aligned} \quad (\text{H.6})$$

$$\begin{aligned} \det \mathbf{M}_{PZ} \cdot D_\beta &= \left[\kappa \widehat{\sigma}(1 - \delta)(1 - \beta) - (\kappa \widehat{\sigma})^2 \right] (\phi - \vartheta) \\ &\quad + \kappa \widehat{\sigma} \phi(1 - q) \left[\beta(1 - \delta) + \delta(1 - \beta p - \beta q + \beta) + \kappa \widehat{\sigma} \right], \end{aligned} \quad (\text{H.7})$$

$$\det \mathbf{M}_{ZZ} \cdot D_\beta = \left[-(\kappa \widehat{\sigma})^2 + \kappa \widehat{\sigma}(1 - \delta)(1 - \beta) \right] (-\vartheta), \quad (\text{H.8})$$

where

$$\vartheta \equiv p + q - 1 - \frac{\left[(1 - \delta) + \delta(2 - p - q) \right] (1 - \beta q - \beta p + \beta)}{\kappa \widehat{\sigma}}. \quad (\text{H.9})$$

When $q = 1$, the remainder terms in (H.6) and (H.7) vanish, and the expressions simplify to

$$\det \mathbf{M}_{PP} \cdot D_\beta = \kappa \widehat{\sigma} \left[(1 - \delta)(1 - \beta) + \kappa \widehat{\sigma}(\phi - 1) \right] (\phi - \vartheta), \quad (\text{H.10})$$

$$\det \mathbf{M}_{ZP} \cdot D_\beta = \left[(\kappa \widehat{\sigma})^2(\phi - 1) + \kappa \widehat{\sigma}(1 - \delta)(1 - \beta) \right] (-\vartheta), \quad (\text{H.11})$$

$$\det \mathbf{M}_{PZ} \cdot D_\beta = \left[\kappa \widehat{\sigma}(1 - \delta)(1 - \beta) - (\kappa \widehat{\sigma})^2 \right] (\phi - \vartheta), \quad (\text{H.12})$$

$$\det \mathbf{M}_{ZZ} \cdot D_\beta = \left[-(\kappa \widehat{\sigma})^2 + \kappa \widehat{\sigma}(1 - \delta)(1 - \beta) \right] (-\vartheta), \quad (\text{H.13})$$

with

$$\vartheta \equiv p - \frac{(1 - \delta p)(1 - \beta p)}{\kappa \widehat{\sigma}}. \quad (\text{H.14})$$

Proof of Proposition 1. The proposition is stated for $q = 1$, so we work with the simplified determinants (H.10)–(H.13). Define

$$F \equiv (1 - \delta)(1 - \beta) + \kappa \widehat{\sigma}(\phi - 1), \quad H \equiv (1 - \delta)(1 - \beta) - \kappa \widehat{\sigma},$$

so that (H.10)–(H.13) take the product form

$$\det \mathbf{M}_{PP} \cdot D_\beta = \kappa \widehat{\sigma} F (\phi - \vartheta), \quad \det \mathbf{M}_{ZP} \cdot D_\beta = \kappa \widehat{\sigma} F (-\vartheta),$$

$$\det \mathbf{M}_{PZ} \cdot D_\beta = \kappa \widehat{\sigma} H (\phi - \vartheta), \quad \det \mathbf{M}_{ZZ} \cdot D_\beta = \kappa \widehat{\sigma} H (-\vartheta).$$

Since $\kappa\widehat{\sigma} \neq 0$ is a common factor, the four determinants share the same (nonzero) sign if and only if

$$F \cdot H > 0 \quad \text{and} \quad (\phi - \vartheta)(-\vartheta) > 0, \quad (\text{H.15})$$

where the second condition states that either $\vartheta < 0$ or $\vartheta > \phi$. Given δ and $\widehat{\sigma}$ as in (4), we sign F , H , and ϑ in three parameter regions.

Case 1 ($\widehat{\sigma} < 0$ and $\delta < 1$, IADL): If $\widehat{\sigma} < 0$, then $1 - \lambda\chi < 0$ and hence $\chi > 1/\lambda > 1$, which implies $\delta < 1$. Here $H > 0$ automatically, since $(1 - \delta)(1 - \beta) > 0$ and $-\kappa\widehat{\sigma} > 0$. From (H.14),

$$\vartheta = p - \frac{(1 - \delta p)(1 - \beta p)}{\kappa\widehat{\sigma}} > p \geq 0,$$

since $(1 - \delta p)(1 - \beta p) > 0$ and $\kappa\widehat{\sigma} < 0$; hence $\vartheta < 0$ is impossible, and the second condition in (H.15) requires $\phi < \vartheta$ – the first condition of case (a) of Proposition 1. Given $H > 0$, the first condition in (H.15) requires $F > 0$:

$$(1 - \delta)(1 - \beta) + \kappa\widehat{\sigma}(\phi - 1) > 0 \iff \delta < 1 - \frac{\kappa(\phi - 1)}{1 - \beta}|\widehat{\sigma}|,$$

the second condition of case (a) (binding only when $\phi > 1$). Under these conditions all four determinants are positive; for instance $\det M_{PP} \cdot D_\beta = \kappa\widehat{\sigma} F (\phi - \vartheta)$ is the product of a negative, a positive, and a negative term. ■

Case 2 ($\widehat{\sigma} > 0$, $1 \leq \chi < 1/\lambda$, and $\delta \geq 1$, HANK-C): Here $H < 0$ always, since $(1 - \delta)(1 - \beta) \leq 0$ and $-\kappa\widehat{\sigma} < 0$. Condition (H.15) therefore requires $F < 0$, i.e., $\kappa\widehat{\sigma}(\phi - 1) < (\delta - 1)(1 - \beta)$. For an active rule ($\phi > \phi_{\text{low}}^*$) this fails, since $F < 0$ is equivalent to $\phi < \phi_{\text{low}}^* = 1 + (\delta - 1)(1 - \beta)/(\kappa\widehat{\sigma})$; at the values of Table 1, $\kappa\widehat{\sigma} \approx 0.13$ while $(\delta - 1)(1 - \beta)$ is of order 10^{-3} or below, so ϕ_{low}^* barely exceeds one. Hence *HANK-C with an active Taylor rule does not possess a unique MSV solution*, and neither case of Proposition 1 applies in this region – consistent with the message of the main text. A passive rule ($\phi < 1$) delivers $F < 0$ automatically; the second condition in (H.15) then additionally requires $\vartheta > \phi$ or $\vartheta < 0$. For $p > 1/\delta$ (e.g. p close to one), $(1 - \delta p) < 0$ and $\kappa\widehat{\sigma} > 0$ imply $\vartheta > p$,¹ so $\vartheta > \phi$ holds for any $\phi \leq p$; for $p < 1/\delta$ small, $\vartheta < 0$ obtains when $\kappa\widehat{\sigma}p < (1 - \delta p)(1 - \beta p)$. In either configuration uniqueness obtains only under passive policy, which violates the

1. Note the direction: $(1 - \delta p)(1 - \beta p) < 0$ divided by $\kappa\widehat{\sigma} > 0$ is negative, and it enters ϑ with a minus sign. At the Gornemann-Kuester-Nakajima calibration of Table 1 with $p = 1$, $\vartheta = 1.0049$.

Blanchard-Kahn conditions in this region – the Catch-22 discussed in §2.1. ■

Case 3 ($\widehat{\sigma} > 0$, $\chi < 1$, and $\delta < 1$): This is the parameter region of case (b) of Proposition 1. Here

$$H > 0 \iff \delta < 1 - \frac{\kappa\widehat{\sigma}}{1-\beta},$$

which is the second condition of case (b). Given $\widehat{\sigma} > 0$ and $H > 0$, we have $F = H + \kappa\widehat{\sigma}\phi > 0$, so $F \cdot H > 0$ holds. With $\delta < 1$, $(1 - \delta p)(1 - \beta p) > 0$ and $\kappa\widehat{\sigma} > 0$ imply $\vartheta < p$; an active rule has $\phi > 1 \geq p > \vartheta$, so $(\phi - \vartheta) > 0$ and the second condition in (H.15) requires $\vartheta < 0$ – the first condition of case (b), which holds if and only if $\kappa\widehat{\sigma}p < (1 - \delta p)(1 - \beta p)$.² Under the conditions of case (b) all four determinants are positive, since $\kappa\widehat{\sigma} > 0$, $F > 0$, $H > 0$, $(\phi - \vartheta) > 0$, and $(-\vartheta) > 0$. ■

H.2 Robust real rate rule

We now examine existence and uniqueness of an MSV solution under the Robust Real Rate rule (13). In the constrained regime ($i_t = -\mu$) the system is identical to the Taylor-rule case,

$$x_t = \delta \mathbb{E}_t x_{t+1} + \widehat{\sigma}(\mu + \mathbb{E}_t \pi_{t+1}) + \varepsilon_t, \quad \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t. \quad (\text{H.16})$$

The unconstrained regime, however, imposes a *different equation*: the rule itself, $\mathbb{E}_t \pi_{t+1} = \phi \pi_t$, with the nominal rate determined residually by the DISE. This matters for the method. Under the Taylor rule, all four regimes share the single reduced IS equation (H.2), in which the max operator merely switches a coefficient; the gap function is therefore continuous piecewise-linear, as the GLM theorem requires. Under the RR, stacking the regime-specific systems does *not* produce the piece-Jacobians of any continuous piecewise-linear map, and a same-sign test on the resulting determinants has no GLM foundation. We therefore construct the continuity-valid formulation explicitly.

Complementarity (gap) formulation. Substituting $r_t = i_t - \mathbb{E}_t \pi_{t+1}$ into (13) gives the fixed-point condition $i_t = \max\{-\mu, i_t - a_t\}$, where $a_t \equiv \mathbb{E}_t \pi_{t+1} - \phi \pi_t$. Its solutions are characterised by a complementarity condition: either the rule binds, $a_t = 0$, with $i_t \geq -\mu$;

2. At $p = 1$ the two conditions of case (b) coincide: $\vartheta < 0 \iff (1 - \delta)(1 - \beta) > \kappa\widehat{\sigma} \iff \delta < 1 - \kappa\widehat{\sigma}/(1 - \beta)$. For $p < 1$ the second implies the first but not conversely: if $H > 0$ and $p > 0$, then $(1 - \delta p)(1 - \beta p) - \kappa\widehat{\sigma}p > (1 - \delta p)(1 - \beta p) - p(1 - \delta)(1 - \beta) = (1 - p)(1 - \delta\beta p) \geq 0$, and the left-hand side equals one at $p = 0$, so $\vartheta < 0$ in either case. Case (b) therefore reduces to $H > 0$.

or the floor binds, $i_t = -\mu$, with $-\mu \geq -\mu - a_t$, i.e. $a_t \geq 0$. Both cases – and no others – are captured by the single continuous equation

$$\min \{ \mathbb{E}_t \pi_{t+1} - \phi \pi_t, i_t + \mu \} = 0, \quad i_t = \widehat{\sigma}^{-1} (\delta \mathbb{E}_t x_{t+1} - x_t + \varepsilon_t) + \mathbb{E}_t \pi_{t+1}, \quad (\text{H.17})$$

with i_t backed out of the DISE. Note the ELB-consistency requirement embedded in (H.17): $\mathbb{E}_t \pi_{t+1} \geq \phi \pi_t$, a restriction on the *sign* of expected disinflation that does not involve μ at all. Under the Taylor rule the analogous requirement is $\phi \pi_t \leq -\mu$, which demands deflation of order μ/ϕ . This difference drives all the results below.

Stacking the two Markov states, the MSV solutions are exactly the zeros of the continuous piecewise-linear map $G(x, \pi)$ whose rows are, for each state j , the complementarity condition (H.17) and the NKPC. Its four linear pieces correspond to the regime configurations $s \in \{PP, ZP, PZ, ZZ\}$, and the GLM theorem applies to G .

Piece determinants. Eliminating x with the regime-independent NKPC rows, $x = \kappa^{-1}(I - \beta K)\pi$, gives $\det J_s = \kappa^2 \det R_s$, where row j of R_s is row j of $(K - \phi I)$ if state j is unconstrained, and row j of $-(\kappa \widehat{\sigma})^{-1} Q$ if it is at the ELB, with Q as in (H.4). Writing $\rho \equiv p + q - 1$ and $\gamma \equiv \beta + \delta - \beta \delta(p + q) + \kappa \widehat{\sigma}$ (so that $Q_{12} = -(1 - p)\gamma$ and $Q_{21} = -(1 - q)\gamma$), the four piece determinants are

$$\det J_{PP} = \kappa^2 (\phi - 1)(\phi - \rho), \quad (\text{H.18})$$

$$\det J_{ZP} = \frac{\kappa}{\widehat{\sigma}} [(\phi - q)Q_{11} - (1 - p)(1 - q)\gamma], \quad (\text{H.19})$$

$$\det J_{PZ} = \frac{\kappa}{\widehat{\sigma}} [(\phi - p)Q_{22} - (1 - p)(1 - q)\gamma], \quad (\text{H.20})$$

$$\det J_{ZZ} = \frac{\det Q}{\widehat{\sigma}^2}, \quad (\text{H.21})$$

where the step-by-step algebra is provided in §D and the expressions are verified numerically against the literal piece Jacobians of G (maximum relative error of order 10^{-12} over the full parameter grid). Two remarks: the expressions are free of the $(\beta\phi - 1)$ singularity, so no auxiliary assumption on $\beta\phi$ is needed; and $\det J_{ZZ} \propto \det Q \propto \det M_{ZZ}$, as it must be, since the fully constrained regime is rule-independent.

The same-sign condition fails under IADL and under standard demand with $H < 0$. At $q = 1$, $Q_{11} = (1 - \delta p)(1 - \beta p) - \kappa \widehat{\sigma} p$, $Q_{22} = (1 - \delta)(1 - \beta) - \kappa \widehat{\sigma} = H$, and $\det Q = Q_{11}Q_{22}$,

so

$$\det J_{PP} = \kappa^2(\phi-1)(\phi-p), \quad \det J_{ZP} = \frac{\kappa(\phi-1)}{\widehat{\sigma}} Q_{11}, \quad \det J_{PZ} = \frac{\kappa(\phi-p)}{\widehat{\sigma}} H, \quad \det J_{ZZ} = \frac{Q_{11}H}{\widehat{\sigma}^2}. \quad (\text{H.22})$$

Under IADL ($\widehat{\sigma} < 0, 0 < \delta < 1$) with $\phi > 1$, both $H > 0$ and $Q_{11} > 0$, so the sign pattern is $\{+, -, -, +\}$. Under standard demand ($\widehat{\sigma} > 0$), $H < 0$ at every calibration of Table 1, so the pattern is $\{+, \text{sgn } Q_{11}, -, -\text{sgn } Q_{11}\}$, both vectors in the order PP, ZP, PZ, ZZ of (H.22). In either case the same-sign condition fails: *with an active RR, the GLM uniqueness condition fails under IADL and under standard demand with $H < 0$ (and numerically for all $(p, q) \in [0, 1]^2$, not only $q = 1$). For $\widehat{\sigma} > 0$ with $H > 0$ all four determinants turn positive and uniqueness obtains; see the footnote to item 3 of Proposition 2.*

Proof of Proposition 2. *Item 1.* In the unconstrained regime the system in (x_t, π_t) , after backing i_t out of the DISE, is $\mathbb{E}_t \pi_{t+1} = \phi \pi_t$ and $\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t$. The pencil has generalised eigenvalues $\{\phi, \infty\}$ (x_t appears only contemporaneously), so the Blanchard-Kahn conditions hold if and only if $\phi > 1$ – independently of $(\delta, \widehat{\sigma})$ and of the shock process. ■

Items 2 and 3. Since the same-sign condition fails, we characterise the solution set by direct enumeration at $q = 1$. Each state is unconstrained (P) or at the ELB (Z); solving the four resulting linear systems exactly (algebra in §D), with Q_{11} and H evaluated at $q = 1$:

- **(P,P):** $\pi_j = x_j = 0$ in both states (the insulated solution), $i_2 = 0, i_1 = \varepsilon^T / \widehat{\sigma}$. Valid iff $\varepsilon^T / \widehat{\sigma} \geq -\mu$; under IADL, iff $\varepsilon^T \leq \bar{\varepsilon}_1 \equiv |\widehat{\sigma}| \mu$.
- **(Z,P):** $\pi_2 = x_2 = 0, i_1 = -\mu, \pi_1 = \kappa(\widehat{\sigma} \mu + \varepsilon^T) / Q_{11}, x_1 = (1 - \beta p) \pi_1 / \kappa$. Valid iff $\phi \pi_1 \leq \mathbb{E}_1 \pi' = p \pi_1$, i.e. $(\phi - p) \pi_1 \leq 0$. Under IADL $Q_{11} > 0$, so for $\phi > p$ validity is $\pi_1 \leq 0 \iff \varepsilon^T \leq \bar{\varepsilon}_1$ – the same region in which (P,P) is valid, rather than its complement.³
- **(P,Z):** state 2 is an absorbing liquidity trap, $\pi_2 = \kappa \widehat{\sigma} \mu / H < 0, x_2 = (1 - \beta) \pi_2 / \kappa$; state 1 satisfies the rule, $\pi_1 = (1 - p) \pi_2 / (\phi - p), x_1 = (1 - \beta \phi) \pi_1 / \kappa$, with i_1 backed out of the DISE. State-2 validity is $(\phi - 1) \pi_2 \leq 0$, which holds for any $\phi > 1$ since $\widehat{\sigma} < 0$ and

3. At $\varepsilon^T = 0$ the validity condition is $Q_{11} / (\kappa \widehat{\sigma}) \leq 0$, which is precisely the existence condition for absorbing ELB equilibria derived by Holden (2024, Supplemental Appendix K.8), with his persistence parameter mapped to our p . Under IADL it holds identically.

$H > 0$ imply $\pi_2 < 0$. State-1 validity $i_1 \geq -\mu$ holds, under IADL, iff $\varepsilon^T \leq \bar{\varepsilon}_2$, where

$$\bar{\varepsilon}_2 \equiv |\widehat{\sigma}|\mu + \frac{(1-p)\pi_2}{\kappa(\phi-p)} \left[(1-\delta p)(1-\beta\phi) - \delta(1-\beta)(\phi-p) - \kappa\widehat{\sigma}\phi \right]. \quad (\text{H.23})$$

Equivalently, $\bar{\varepsilon}_2 = |\widehat{\sigma}|\mu(\phi-1)Q_{11}/[(\phi-p)H]$, which is positive under IADL with $\phi > 1$, since $Q_{11} > 0$ and $H > 0$; and $\bar{\varepsilon}_2 > \bar{\varepsilon}_1$ if and only if $(\phi-1)Q_{11} > (\phi-p)H$.

- **(Z,Z):** state 2 as in (P,Z); state 1 at the ELB with validity $\phi\pi_1 \leq p\pi_1 + (1-p)\pi_2$. The (Z,Z) and (P,Z) solutions coincide exactly where state 1 sits at the kink of the rule ($i_1 = -\mu$ and $\phi\pi_1 = \mathbb{E}_1\pi'$ simultaneously). Both validity conditions are affine in ε^T and, under IADL with $\phi > p$, decreasing: i_1 has slope $1/\widehat{\sigma} < 0$ in (P,Z), and the state-1 slack $p\pi_1 + (1-p)\pi_2 - \phi\pi_1$ has slope $(p-\phi)\kappa/Q_{11} < 0$ in (Z,Z). The two configurations are therefore valid on the same half-line $\varepsilon^T \leq \bar{\varepsilon}_2$.

Counting: all four configurations are valid for $\varepsilon^T < \min\{\bar{\varepsilon}_1, \bar{\varepsilon}_2\}$, exactly one pair survives between the thresholds, and none is valid for $\varepsilon^T > \max\{\bar{\varepsilon}_1, \bar{\varepsilon}_2\}$. Since $\bar{\varepsilon}_1 = |\widehat{\sigma}|\mu > 0$, the multiplicity region contains $\varepsilon^T = 0$ and every deflationary shock; the MSV solution is unique only at $\varepsilon^T = \max\{\bar{\varepsilon}_1, \bar{\varepsilon}_2\}$ and never on an interval.⁴ This proves item 2.

For item 3, uniqueness for every realisation of ε^T requires (i) the trap configurations to be inadmissible and (ii) the (P,P) and (Z,P) validity regions to partition the shock space. From the conditions above, (i) holds iff $\phi < 1$ (then $(\phi-1)\pi_2 \leq 0$ requires $\pi_2 \geq 0$, contradicting $\pi_2 < 0$; this step uses $\pi_2 < 0$, i.e. IADL or standard demand with $H < 0$, and for $\widehat{\sigma} > 0$ with $H > 0$ one has $\pi_2 = \kappa\widehat{\sigma}\mu/H > 0$ instead, so that the trap configurations are inadmissible under an *active* rule and the enumeration delivers exactly one MSV solution for every ε^T), and (ii) holds iff $\phi < p$ (then $(\phi-p)\pi_1 \leq 0$ selects $\pi_1 \geq 0 \iff \varepsilon^T \geq \bar{\varepsilon}_1$, the exact complement of the (P,P) region, with the two solutions coinciding at the boundary). Hence the MSV solution is unique for every ε^T if and only if $\phi < p$ – and by item 1 any such ϕ violates the Blanchard-Kahn conditions. ■

Remark (scope of the continuity requirement). The continuity issue is specific to the RR's regime structure, in which the two regimes impose different equations. Under the

4. At an illustrative IADL calibration ($\lambda = 0.6$, $\chi = 2.5$, hence $\delta = 0.88$ and $\widehat{\sigma} = -0.4$, with $\phi = 2$, $p = 0.9$, $\mu = 0.02$): $\bar{\varepsilon}_1 = 0.0080$ and $\bar{\varepsilon}_2 = 0.0087$. Numerically $\bar{\varepsilon}_2 > \bar{\varepsilon}_1$ at most, but not all, IADL calibrations; the proposition does not rely on the ordering. The thresholds scale linearly with μ , so no $\mu > 0$ restores uniqueness. At each threshold the two configurations that become inadmissible just above it coincide, so the distinct-solution count is three at the lower threshold and one at the upper, whichever way $\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$ are ordered.

Taylor rule and under yield-curve control the max operator enters a single shared equation – (H.2) and its YCC analogue – so the gap functions used in §H.1 and §H.3 are continuous and the GLM theorem applies to them. Propositions 1 and 3 are unaffected.

H.3 Yield-curve control

We verify existence and uniqueness of an MSV solution for the HANK model under always-on yield-curve control. The aggregate IS (15) combines the standard short-rate Taylor rule $i_t = \max\{-\mu, \phi\pi_t\}$ with the long-rate price-peg $i_t^L = \phi_L\pi_t$ at segmentation weight ω_r .

Two features of this microfoundation deserve emphasis. First, the r -type Euler equation prices the consol at its current yield i_t^L . This follows the hold-to-maturity treatment of long-term debt in Chen, Cúrdia, and Ferrero (2012): restricted savers face prohibitive transaction costs in secondary-market trading, so the expected capital-gain term $\mathbb{E}_t i_{t+1}^L$ that would enter a one-period holding-return Euler equation is absent from their intertemporal margin.⁵ Second, aggregation preserves the coefficients $(\widehat{\sigma}, \delta)$ of (4) exactly because segmentation reallocates a fixed share ω_r within the saver group: the saver/HtM composition, the idiosyncratic switching probabilities (common across u - and r -types), and hence the weights of the Bilbiie aggregator are all unchanged, and ω_r enters the aggregate system only through the composite intertemporal price $(1 - \omega_r)i_t + \omega_r i_t^L$.

Matrix formulation. Stacking (15) across the two Markov states in vector form and substituting the rules:

$$x = \delta Kx - \widehat{\sigma}[\Phi_s \pi - K\pi] + \varepsilon_s, \quad \Phi_s \equiv \text{diag}(\phi_{s,1}^{\text{eff}}, \phi_{s,2}^{\text{eff}}),$$

where the regime-specific disturbance ε_s absorbs the intercept $+\widehat{\sigma}(1 - \omega_r)\mu$ in any state whose short rate is at the ELB. Substituting $\pi = Cx$ with $C = \kappa(I - \beta K)^{-1}$ and right-multiplying by $(I - \beta K)$:

$$M_s^{\text{YCC}}(I - \beta K) = (I - \delta K)(I - \beta K) - \widehat{\sigma}\kappa(K - \Phi_s) = Q + \widehat{\sigma}\kappa\Phi_s, \quad (\text{H.24})$$

where $Q \equiv (I - \delta K)(I - \beta K) - \widehat{\sigma}\kappa K$ is the standard Taylor-rule Q -matrix from (H.4). Unlike the long-rate rules of §H.4, the regime distinction enters as a *diagonal* perturbation of Q

5. With mark-to-market trading, the one-period holding return on the consol would carry a duration-amplified capital-gain term in $\mathbb{E}_t i_{t+1}^L$; the hold-to-maturity friction is precisely what shuts this channel off and licenses the yield-based Euler equation as a primitive.

that keeps a policy coefficient, $\omega_r \phi_L$, in a state at the ELB, and the consol recursion plays no role.

General- (p, q) determinant. Taking det of (H.24):

$$\det \mathbf{M}_s^{YCC} \cdot D_\beta = \det(\mathbf{Q} + \widehat{\sigma} \kappa \boldsymbol{\Phi}_s) = (Q_{11} + \widehat{\sigma} \kappa \phi_{s,1}^{\text{eff}})(Q_{22} + \widehat{\sigma} \kappa \phi_{s,2}^{\text{eff}}) - Q_{12}Q_{21}. \quad (\text{H.25})$$

This closed form holds at any (p, q) ; the four regimes differ only in the values of $\phi_{s,1}^{\text{eff}}$ and $\phi_{s,2}^{\text{eff}}$ assigned per state by (17).

Four-sign factorisation at $q = 1$. At $q = 1$, $\mathbf{K}$ is upper triangular and so is $\mathbf{Q}$, with $Q_{21} = 0$ and the diagonal entries

$$Q_{11}|_{q=1} = (1 - \delta p)(1 - \beta p) - \widehat{\sigma} \kappa p = \mathcal{E}_p(0), \quad Q_{22}|_{q=1} = (1 - \delta)(1 - \beta) - \widehat{\sigma} \kappa = \mathcal{E}_1(0),$$

where

$$\mathcal{E}_\mu(\phi) \equiv (1 - \delta \mu)(1 - \beta \mu) + \widehat{\sigma} \kappa (\phi - \mu) \quad (\text{H.26})$$

is the standard TR-style eigenvalue factor. The determinant then collapses to a product:

$$\det \mathbf{M}_{PP}^{YCC} \cdot D_\beta|_{q=1} = \mathcal{E}_p(\bar{\phi}) \cdot \mathcal{E}_1(\bar{\phi}), \quad \det \mathbf{M}_{PZ}^{YCC} \cdot D_\beta|_{q=1} = \mathcal{E}_p(\bar{\phi}) \cdot \mathcal{E}_1(\omega_r \phi_L), \quad (\text{H.27})$$

$$\det \mathbf{M}_{ZP}^{YCC} \cdot D_\beta|_{q=1} = \mathcal{E}_p(\omega_r \phi_L) \cdot \mathcal{E}_1(\bar{\phi}), \quad \det \mathbf{M}_{ZZ}^{YCC} \cdot D_\beta|_{q=1} = \mathcal{E}_p(\omega_r \phi_L) \cdot \mathcal{E}_1(\omega_r \phi_L), \quad (\text{H.28})$$

with $\bar{\phi} = (1 - \omega_r)\phi + \omega_r \phi_L$ from (17). The four determinants are four distinct products of the eigenvalue factors at $\mu \in \{p, 1\}$ evaluated at the active-state coefficient $\bar{\phi}$ or the ELB coefficient $\omega_r \phi_L$.

Proof of Proposition 3 (at $q = 1$). The GLM theorem requires

$$\text{sgn} \det \mathbf{M}_{PP}^{YCC} = \text{sgn} \det \mathbf{M}_{PZ}^{YCC} = \text{sgn} \det \mathbf{M}_{ZP}^{YCC} = \text{sgn} \det \mathbf{M}_{ZZ}^{YCC}.$$

Via (H.27)–(H.28), this reduces to: $\text{sgn} \mathcal{E}_\mu(\bar{\phi}) = \text{sgn} \mathcal{E}_\mu(\omega_r \phi_L)$ for both $\mu \in \{1, p\}$. Since each $\mathcal{E}_\mu$ is linear in ϕ with slope $\widehat{\sigma} \kappa$, the same-sign condition is equivalent to: both $\bar{\phi}$ and $\omega_r \phi_L$ lying strictly above the eigenvalue zero $\phi_\mu^* = \mu - (1 - \delta \mu)(1 - \beta \mu)/(\widehat{\sigma} \kappa)$, or both lying strictly below it, for each μ .

Non-IADL case ($\widehat{\sigma} > 0$). Each $\mathcal{E}_\mu$ is increasing in ϕ . The intercept $\mathcal{E}_\mu(0) = (1 - \delta\mu)(1 - \beta\mu) - \widehat{\sigma}\kappa\mu$ is negative for plausible HANK calibrations (the second term dominates by orders of magnitude at $\beta \approx 1$), so $\phi_\mu^* > 0$. The same-sign condition is satisfied with both factors positive iff $\min(\bar{\phi}, \omega_r\phi_L) > \max(\phi_1^*, \phi_p^*) \approx 1$. Since $\omega_r\phi_L \leq \bar{\phi}$, the binding constraint is $\omega_r\phi_L > \max(\phi_1^*, \phi_p^*)$; at the calibrations of Table 1 the max is $\phi_1^* \approx 1$, equivalently (22). There $\phi_p^* < \phi_1^* = \phi_{\text{low}}^*$, so every other sign-consistent configuration places $\bar{\phi}$ below ϕ_{low}^* and violates the Blanchard-Kahn conditions.

IADL case ($\widehat{\sigma} < 0, \delta < 1$). Each $\mathcal{E}_\mu$ is decreasing in ϕ with $\mathcal{E}_\mu(0) > 0$. The four-sign condition holds iff $\bar{\phi}$ and $\omega_r\phi_L$ lie on the same side of ϕ_1^* and on the same side of ϕ_p^* . Two branches matter: (i) both factors positive when $\max(\bar{\phi}, \omega_r\phi_L) = \bar{\phi} < \min(\phi_1^*, \phi_p^*)$, which is $\bar{\phi} < \phi_1^* \approx 1$ when $\phi_1^* \leq \phi_p^*$, as at the illustrative calibration of Online Appendix Figure E.1; and (ii) both factors negative when $\min(\bar{\phi}, \omega_r\phi_L) = \omega_r\phi_L > \max(\phi_1^*, \phi_p^*)$. The branches are not symmetric once local determinacy is imposed. The trace-root condition of the Blanchard-Kahn criterion (Online Appendix §B.6) reads $\bar{\phi} < \phi_1^* = 1 + (1 - \delta)(1 - \beta)/(\kappa|\widehat{\sigma}|)$ under IADL – the analogue of the passive-policy requirement of Bilbiie (2008) – and is necessary for local determinacy at all but very aggressive responses. Branch (i) satisfies it; branch (ii), with $\bar{\phi} > \phi_1^*$, therefore satisfies the four-sign (uniqueness) condition but violates the Blanchard-Kahn conditions unless $\bar{\phi}$ reaches the second, very aggressive interval of §B.6 (on which $D < 0$), which lies far outside plausible policy (it begins at $\bar{\phi} \approx 52$ at $\delta = 0.88, \widehat{\sigma} = -0.4$). At plausible responses the globally determinate IADL configuration is branch (i), the passive branch. ■

Numerical verification. The general- (p, q) closed form (H.25) and the $q = 1$ factorisation (H.27)–(H.28) are verified against direct construction of M_s^{YCC} from (15) in Julia/verify_YCC_determinants.jl over 38 400 parameter combinations; the maximum relative error is below 10^{-10} across all four regimes. The AD-AS slopes used in Online Appendix Figure E.1 are independently verified in Julia/verify_AD_YCC.jl.

H.4 Long-rate rules without segmentation

A natural alternative to YCC is to set a feedback rule on a long-term real or nominal interest rate without bond-market segmentation. We collect here the qualitative analysis of these specifications and their failure outside IADL; the detailed determinant algebra appears in §F and §G.

Long-term nominal rate rule (nLRR). Let i_t^L denote the yield on a nominal consol satisfying $i_t^L = (1 - \beta)i_t + \beta\mathbb{E}_t i_{t+1}^L$, with policy rule $i_t^L = \phi_\pi \pi_t$. In a representative-agent bond market all households' Euler equations face the same effective rate, so the consol recursion eliminates the short rate i_t in favour of π_t through the NKPC identity. The unconstrained PP regime delivers a clean eigenvalue factorisation $\det \mathbf{M}_{PP}^{nLRR} \cdot D_\beta = \mathcal{E}_1^{nLRR}(\phi_\pi) \cdot \mathcal{E}_\rho^{nLRR}(\phi_\pi)$, where $\rho \equiv p + q - 1$ denotes the second eigenvalue of the Markov kernel $\mathbf{K}$ (the first being unity), and

$$\mathcal{E}_\mu^{nLRR}(\phi) \equiv (1 - \delta\mu)(1 - \beta\mu) + \widehat{\sigma}\kappa \left[\frac{\phi(1 - \beta\mu)}{1 - \beta} - \mu \right]. \quad (\text{H.29})$$

At $q = 1$ the four-regime factorisation mirrors (H.27)–(H.28) but with $\mathcal{E}_\mu^{nLRR}$ replacing $\mathcal{E}_\mu$ and the policy argument equal to ϕ_π when the rule is active and 0 when the ELB binds (full derivation in §F).

Failure outside IADL. Under non-IADL ($\widehat{\sigma} > 0$), $\mathcal{E}_\mu^{nLRR}(0) = (1 - \delta\mu)(1 - \beta\mu) - \widehat{\sigma}\kappa\mu < 0$ for plausible calibrations because the intercept $(1 - \delta\mu)(1 - \beta\mu)$, at most of order 10^{-2} , is dominated by $\widehat{\sigma}\kappa\mu \sim 10^{-1}$. The same-sign condition therefore demands $\mathcal{E}_\mu^{nLRR}(\phi_\pi) < 0$, i.e. ϕ_π below the zero of $\mathcal{E}_\mu^{nLRR}$. The consol recursion multiplies ϕ_π by the amplification factor $(1 - \beta\mu)/(1 - \beta)$, which ranges from ≈ 11 at $\mu = \rho = 0.9$ (the $q = 1, p = 0.9$ case) up to $1/(1 - \beta) = 100$ as $\rho \rightarrow 0$; at HANK calibrations the resulting eigenvalue zero is $\phi^{**} \approx 0.066\text{--}0.083$ for $\mu = \rho$ – well below the BK requirement $\phi_\pi > \phi_{\text{low}}^* \approx 1$. The two conditions are mutually exclusive: no ϕ_π satisfies both BK and GLM under non-IADL HANK with the nLRR.

Long-term real rate rule (real LRR). The analogous specification with the *real* consol $r_t^L = (1 - \beta)r_t + \beta\mathbb{E}_t r_{t+1}^L$ and policy rule $r_t^L = \phi_\pi \pi_t$ exhibits a more severe pathology: substituting the rule into the IS through the consol recursion, the NKPC identity $\pi_t - \beta\mathbb{E}_t \pi_{t+1} = \kappa x_t$ causes $\mathbb{E}_t \pi_{t+1}$ to cancel *exactly*, leaving a one-dimensional forward equation $\omega x_t = \delta\mathbb{E}_t x_{t+1} + \varepsilon_t$ with $\omega \equiv 1 + \widehat{\sigma}\kappa\phi_\pi/(1 - \beta)$. The reduced system has no role for inflation expectations or for the short nominal rate, so the ELB constraint $i_t \geq -\mu$ does not bind on any variable that enters the reduced dynamics: it has no purchase on determinacy. The four-regime structure collapses, the GLM sign check becomes vacuous, and the determinacy properties reduce to those of a one-dimensional Sargent-Wallace system. This vacuity is a property of the *direct-implementation* reading, under which the central bank transacts in indexed bonds at the announced real yield and the short nominal rate is

a shadow variable free to breach $-\mu$. If the nominal floor is instead enforced, the bank pegs $i_t = -\mu$ when it binds, expected inflation re-enters the IS through Fisher's identity, and the floor regime becomes a genuinely two-dimensional pegged-rate system distinct from the collapsed one-dimensional active regime; the four-sign condition is then non-vacuous but generically fails. Either way the real LRR is not a substantive resolution of the ELB problem. We treat this as a pedagogical limit rather than a substantive policy specification; §G records the algebra.

Comparison with YCC. Bond-market segmentation is what differentiates YCC from these long-rate rules. In a segmented market the long-rate channel enters the IS with weight ω_r *additively* alongside the short-rate channel, and the consol recursion plays no role in the IS reduction. The eigenvalue factor reverts to the TR form (H.26) – which is linear in ϕ *without* the amplification $(1 - \beta\mu)/(1 - \beta)$ – and the threshold for GLM under non-IADL becomes $\omega_r\phi_L > 1$, comfortably compatible with the BK requirement on the active short-rate channel.

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