Notes on (T)HANK*

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1 Aggregate Demand and the (New) Keynesian Cross

In what follows, I stick to the derivation of Bilbiie – but I make some changes to notation and timing to keep things consistent with my other notes.

Think back to your undergraduate macroeconomics courses, in particular to the Keynesian cross – remember the discussion about inventories, leakages, injections, and so on. Let's postulate a consumption function or a planned expenditure (PE) curve:

$$C = C(\underset{+}{Y}, \underset{-}{r}),$$

where consumption is increasing in income (Y) and decreasing in the real interest rate (r). Thus we can write $C_Y \in (0, 1)$ and $C_r < 0$. With an equilibrium condition, the economy-wide resource constraint (ERC), we then have that consumption is equal to income.

As set out in Bilbiie (2020): The slope of the PE curve is the marginal propensity to consume (MPC) out of income, i.e., C_Y. Meanwhile, a cut in real interest rates leads to an increase in consumption, i.e., C_r, which is the autonomous expenditure increase. But there is an additional increase – the equilibrium level of consumption and output increase. This is the famous Keynesian multiplier effect:

$$C_R \left(1 + C_Y + C_Y^2 + ...\right).$$

The initial increase in consumption and income leads to a second round increase in consumption by the MPC. This leads to an increase in income, and a third round increase in consumption by the MPC. So on and so forth. The equilibrium increase in consumption and income is therefore

$$dC = dY = \frac{C_r}{1 - C_Y} d(-r),$$

*These notes are based on the amazing work of Bilbiie (2008, 2020, 2021) – all of which are recommended reads.

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where $C_r/(1-C_Y)$ is the multiplier.

Figure 1 is a plot of Bilbiie's New Keynesian cross, which embeds the discussion above and the mechanism of the classical Keynesian cross. Denote the MPC as $C_Y = \omega$, Ω as the multiplier (equilibrium) increase of an interest rate cut, the autonomous expenditure increase $C_r = \Omega_D$, and the indirect or residual equilibrium increase as Ω_I.

Bilbiie assumes an environment of sticky prices and whereby output is demand-determined – so abstract from any supply side factors. The real interest rate is targeted by the central bank so that prices are fixed (Bilbiie, 2008). In other words, nominal interest rates are set to neutralise changes in expected inflation in the Fisher equation,

$$\hat{R}_t = \mathbb{E}_t \hat{\pi}_{t+1} + \hat{r}_t,$$

with $\hat{r}_t = \bar{r}_t$ which is what we mean by the real rate being targeted.

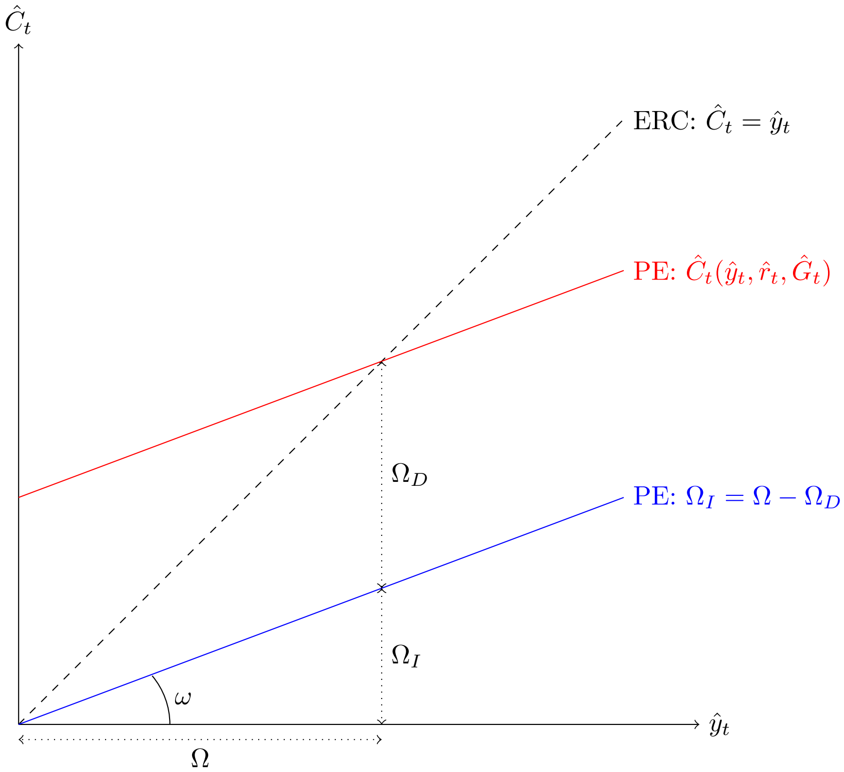

Figure 1: The New Keynesian Cross

A few more remarks about Figure 1 to familiarise the notation: The key equation that Bilbiie derives in his analysis is consumption or aggregate demand (AD) as a function of current income for a given real interest rate,

$$\hat{C}_t = \omega \hat{y}_t - \Omega_D \hat{r}_t. \tag{1}$$

To restate things, ω is the slope of PE and is the aggregate MPC. Ω_D is the shift of the PE curve, and is the autonomous expenditure change when the interest rate is cut. Without capital or inventories, this is simply intertemporal substitution: $\hat{r}_t \downarrow$ with incomes given, agents want to bring consumption forward. Without assets to liquidate or "disinvest" their income adjusts to deliver equilibrium. Ω is the Keynesian multiplier – the equilibrium effect of a cut in interest rates on AD and income, given as

$$\Omega = \frac{\Omega_D}{1-\omega}.$$

So a reduction in the interest rate results in PE curve (with slope ω) shifting upwards by Ω_D, and equilibrium moves from the origin, $\hat{C}_t = \hat{y}_t = 0$, to Ω.

1.1 The New Keynesian Cross with RANK

Consider an environment with full information rational expectations (FIRE), competitive labour markets, and complete financial markets. In such a setup, an agent would maximise his or her lifetime utility,

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u(C^i_{t+s}, L^i_{t+s}),$$

subject to their flow nominal budget constraint,

$$P_t C^i_t + A^i_t + V_t \psi^i_t \leq W_t L^i_t + B^i_{t-1} + \psi^i_{t-1}(V_t + P_t D_t),$$

where A^i_t is the agent's nominal end of period t portfolio of all state-contingent assets (except equity), B^i_{t-1} is the start of period wealth, and ψ^i_t are equity held by the agent.

Assuming a no-arbitrage condition implies that there exists a pricing kernel or stochastic discount factor (SDF), $\Lambda^i_{t,t+1}$, such that:

$$\frac{A^i_t}{P_t} = \mathbb{E}_t \Lambda^i_{t,t+1} \frac{B^i_t}{P_{t+1}},$$
$$\frac{V_t}{P_t} = \mathbb{E}_t \Lambda^i_{t,t+1} \left(\frac{V_{t+1}}{P_{t+1}} + D_{t+1} \right),$$

and whereby the gross real interest rate is pinned down by

$$\frac{1}{r_t} = \mathbb{E}_t \Lambda^i_{t,t+1}. \tag{2}$$

With a no-arbitrage condition and the ability for agents to accumulate wealth, we assume that agents hold a constant fraction of shares (i.e., we have no trade such that $\psi^i_t = \psi^i, \forall t$). Then, define the period income for an agent as

$$Y^i_t \equiv \psi^i D_t + \frac{W_t}{P_t} L^i_t.$$

With these assumptions, we can then rewrite the intertemporal budget constraint as the following:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \Lambda^i_{t,t+s} C^i_{t+s} \leq \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda^i_{t,t+s} Y^i_{t+s}. \tag{3}$$

Obtaining the FOCs by maximising $u(\cdot)$ gives us an expression for the Euler equation,

$$\beta \mathbb{E}_t \frac{u_C(C^i_{t+1})}{u_C(C^i_t)} = \mathbb{E}_t \Lambda^i_{t,t+1}.$$

Putting things together, we have the Euler equation, that the transversality condition holds,

$$\lim_{s\to\infty} \mathbb{E}_t \Lambda^i_{t,t+s} A^i_{t+s} = \lim_{s\to\infty} \mathbb{E}_t \Lambda^i_{t,t+s} V_{t+s} = 0,$$

and that the budget constraint (either the flow or the intertemporal budget constraint) binds with equality. Substitute the definition of the gross real rate from the no-arbitrage condition (2) into the Euler equation to get the following familiar expression,

$$\frac{1}{R_t} = \beta \mathbb{E}_t \frac{u_C(C^i_{t+1})}{u_C(C^i_t)}. \tag{4}$$

Next, we need to do some log-linearisation. Start off by writing (3) as

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(\hat{\Lambda}^i_{t,t+s} + \hat{C}^i_{t+s} \right) = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(\hat{\Lambda}^i_{t,t+s} + \hat{Y}^i_{t+s} \right). \tag{5}$$

To get this, first note that we assume standard constant relative risk aversion (CRRA) preferences:

$$u\left(C^i_t, L^i_t\right) = \frac{(C^i_t)^{1-\sigma}}{1-\sigma} - \varphi_0 \frac{(L^i_t)^{1+\varphi}}{1+\varphi}, \tag{6}$$

where σ is the coefficient of relative risk aversion given by $\sigma = -\frac{u_{CC}}{u_C} C$ (the "curvature of consumption"), and φ is the inverse-Frisch elasticity of labour supply. Thus we have $u_C = C^{-\sigma}$ (and $u_{CC} = -\sigma / C^{1+\sigma}$), and we can write:

$$\begin{aligned} \mathbb{E}_t \hat{\Lambda}^i_{t,t+s} &= s \ln \beta + \mathbb{E}_t \ln u_C\left(C^i_{t+s}\right) - \ln u_C\left(C^i_t\right) - s \ln \beta \\ &= \sigma \left(\ln C^i_t - \mathbb{E}_t \ln C^i_{t+s} \right) \\ &= \sigma \left(\hat{C}^i_t - \mathbb{E}_t \hat{C}^i_{t+s} \right). \end{aligned} \tag{7}$$

Additionally, use

$$-\hat{r}_t = \mathbb{E}_t \hat{\Lambda}_{t,t+1}$$

to write

$$
\begin{aligned}
\hat{\Lambda}_{t,t+s} &= \hat{\Lambda}_{t,t+1} + \hat{\Lambda}_{t+1,t+2} + ... + \hat{\Lambda}_{t+s-1,t+s} \\
&= -\sum_{s=0}^{\infty} \hat{r}_{t+s} \\
&= -\sum_{k=0}^{s} \hat{r}_{t+k}.
\end{aligned} \tag{8}
$$

Using these, we can then write (7) as

$$
\hat{C}_t^i = \mathbb{E}_t \hat{C}_{t+1}^i - \frac{1}{\sigma} \mathbb{E}_t \hat{\Lambda}_{t,t+1}^i. \tag{9}
$$

Log-linearising (3) and substituting in (9) gives (5).[1] Then, add $\mathbb{E}_t \sum_{s=0}^{\infty} \left(\sigma^{-1} - 1\right) \hat{\Lambda}_{t,t+s}^i$ to the LHS and RHS, and do a bit of algebra:

$$
\begin{aligned}
\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\hat{\Lambda}_{t,t+s}^i + \hat{C}_{t+s}^i + \left(\frac{1}{\sigma} - 1 \right) \hat{\Lambda}_{t,t+s}^i \right] &= \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\hat{\Lambda}_{t,t+s}^i + \hat{Y}_{t+s}^i + \left(\frac{1}{\sigma} - 1 \right) \hat{\Lambda}_{t,t+s}^i \right] \\
\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{C}_{t+s}^i + \frac{1}{\sigma} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{\Lambda}_{t,t+s}^i &= \frac{1}{\sigma} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{\Lambda}_{t,t+s}^i + \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{Y}_{t+s}^i \\
\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{C}_{t+s}^i + \frac{1}{\sigma} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \sigma \left(\hat{C}_t^i - \hat{C}_{t+s}^i \right) &= \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{Y}_{t+s}^i - \frac{1}{\sigma} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{r}_{t+s} \\
\sum_{s=0}^{\infty} \beta^s \hat{C}_t^i &= \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{Y}_{t+s}^i + \frac{1}{\sigma} \left(0 - \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s \sum_{k=0}^{s} \hat{r}_{t+k} \right) \\
\frac{1}{1-\beta} \hat{C}_t^i &= \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{Y}_{t+s}^i - \frac{\beta}{\sigma(1-\beta)} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{r}_{t+s},
\end{aligned}
$$

to then get

$$
\hat{C}_t^i = (1-\beta) \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{Y}_{t+s}^i - \frac{\beta}{\sigma} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{r}_{t+s}^i.
$$

To write this recursively, first start out by writing

$$
\hat{C}_t^i = (1-\beta) \left(\hat{Y}_t^i + \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s \hat{Y}_{t+s}^i \right) - \frac{\beta}{\sigma} \left(\hat{r}_t + \mathbb{E} \sum_{s=1}^{\infty} \beta^s \hat{r}_{t+s} \right),
$$

and define

$$
\beta \hat{C}_{t+1}^i = (1-\beta) \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{s+1} \hat{Y}_{t+s+1}^i - \frac{\beta}{\sigma} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{s+1} \hat{r}_{t+s+1}.
$$

1. Also use the fact that in steady state $\Lambda_{t,t+s}^i|_{s=0} = 1$ and $\Lambda_{t,t+s}^i = \beta^s$.

Putting things together, you can then write the consumption Euler equation as

$$\hat{C}_t^i = (1-\beta)\hat{Y}_t^i - \frac{\beta}{\sigma}\hat{r}_t + \beta\hat{C}_{t+1}^i. \tag{10}$$

What this equation is saying is that $1-\beta$ is the MPC from a transitory income increase, β is the marginal propensity to save. As Bilbiie states: even though there are no assets to save in, shifts in savings through substitution effects need to be accompanied by compensating income effect shifts in order to have zero equilibrium saving which changes equilibrium income.

We can simplify the above Euler equation further by imposing the market clearing condition:

$$\hat{C}_t^i = \hat{y}_t^i \equiv \hat{Y}_t^i - \hat{T}_t^i,$$

where $\hat{T}_t^i$ are taxes levied by the government. Substituting this into the consumption Euler equation (10) we get

$$\hat{C}_t^i = \mathbb{E}_t\hat{C}_{t+1}^i - \frac{1}{\sigma}\hat{r}_t. \tag{11}$$

We are now ready to obtain an expression like (1) and plot the NK cross for a RANK model. First, get the Keynesian multiplier Ω for a real interest rate cut. We can do this by differentiating $\hat{C}_t^i$ wrt $\hat{r}_t$ – but how do we deal with the expectation operator? Recall that we are operating with FIRE, and make the assumption that an interest rate reduction has persistence of ρ. Thus we can write $\mathbb{E}_t\hat{C}_{t+1}^i = \rho\hat{C}_t^i$. We then get

$$\Omega \equiv \frac{d\hat{C}_t^i}{d\left(-\hat{r}_t\right)} = \frac{1}{\sigma(1-\rho)},$$

from (11). Then to get the direct effect of the interest rate reduction, Ω_D, use (10) and differentiate holding income fixed:

$$\Omega_D \equiv \left.\frac{d\hat{C}_t^i}{d\left(-\hat{r}_t\right)}\right|_{\hat{Y}_t^i=\bar{Y}} = \frac{\beta}{\sigma(1-\beta\rho)}.$$

Also, recall from your microeconomics lectures that the direct effect can be thought of as the substitution effect – and is the notation used as in Auclert (2019).[2] The indirect effect is simply

$$\Omega_I \equiv \left.\frac{d\hat{C}_t^i}{d\left(-\hat{r}_t\right)}\right|_{\hat{r}_t^i=\bar{r}} = \Omega - \Omega_D = \frac{1}{\sigma(1-\rho)} - \frac{\beta}{\sigma(1-\beta\rho)} = \frac{1-\beta}{\sigma(1-\rho)(1-\beta\rho)},$$

and which can be thought of as the income effect. Alternatively, we can think of the direct and indirect effects as the partial and general equilibrium effects, respectively. Meanwhile, the MPC is

$$\omega \equiv \frac{\Omega_I}{\Omega} = \frac{1-\beta}{1-\beta\rho}.$$

2. Theorem 3.

Finally, assuming that fiscal policy is implemented by $\hat{G}_t = \hat{T}_t$, define the fiscal multiplier as

$$\mathcal{M} = \frac{d\hat{y}_t}{d\hat{G}_t} = 1 \Leftrightarrow \frac{d\hat{C}_t}{d\hat{G}_t} = 0.$$

Putting everything together, we can write the RANK Euler equation for the NK cross as

$$\hat{C}_t = \omega\hat{y}_t - (1-\omega)\Omega\hat{r}_t + (1-\omega)(\mathcal{M}-1)\hat{G}_t,$$

and get to the following:

Proposition (Bilbiie (2020) Proposition 1)**.** *In RANK, the MPC ω, autonomous expenditure increase Ω_D, and multiplier Ω (for an interest rate cut of persistence ρ) are*

$$\omega = \frac{1-\beta}{1-\beta\rho};\ \Omega_D = \frac{\beta}{\sigma(1-\beta\rho)};\ \Omega = \frac{1}{\sigma(1-\rho)}.$$

Now, we can use the NK cross to assess the RANK model. Consider a transitory monetary policy shock ($\rho = 0$), which will clearly show the deficiencies of RANK. The general equilibrium effect of this is $\Omega = \sigma^{-1}$, which according to a long history of macro literature is close to zero (see, for example, Hall (1988), Campbell and Mankiw (1990), Bilbiie and Straub (2012) and Ascari, Magnusson, and Mavroeidis (2021)). The second problem – highlighted by Kaplan, Moll, and Violante (2018) – is that the MPC (share of the indirect effect) becomes $\omega = 1-\beta$. This is, like the general equilibrium effect, close to zero for high values of β – an observation in line with declining real interest rates over the past few decades. Put simply, RANK fails to generate both a general equilibrium and indirect effect of a monetary policy shock. In fact, even calibrating $\beta = 0.99$ and $\rho = 0.55$, we get an MPC of $\omega = \frac{1-\beta}{1-\beta\rho} = 0.02$. Even setting $\rho = 0.9$ we only get $\omega = 0.09$, which implies that the PE curve is basically flat. Additionally, the problems of fiscal policy in RANK models is obvious: the fiscal multiplier on consumption of public expenditures is 0!

To quote Bilbiie and paraphrase a bit: According to RANK, consumption is almost insensitive to current income, which contradicts evidence obtained using a wide spectrum of micro and macro data. To make matters worse, RANK is, paradoxically, not very "general equilibrium" either – almost all the effect of monetary policy comes from the partial equilibrium (direct) shift of the PE curve! Such considerations spurred the development of TANK and HANK models.

1.2 TANK Models and the New Keynesian Cross

Now move onto a standard two-agent New Keynesian (TANK) to see how it can address some RANK's shortcomings. Assume that there are two types of agents $i \in \{H, S\}$ amongst a unit mass of households. Proportion λ are "hand-to-mouth" households ($i = H$) and consume all of their income, and thus do not participate in asset markets:

$$C_t^H = W_t L_t^H + \mathcal{T}_t^H,$$

where $\mathcal{T}_t^i$ are fiscal transfers to type i households. The remaining $1-\lambda$ proportion of households are savers ($i = S$). They can save in state-contingent securities as well as equity in monopolistically competitive firms.[3] Thus, they receive both profits and labour income, and their budget constraint is:

$$C_t^S + A_t^S + V_t\psi_t^S \leq W_tL_t^S + B_t^S + \psi_{t-1}^S\left(V_t + P_tD_t\right) + \mathcal{T}_t^S.$$

Following the logic we had in Section 1.1, asset markets clear, equity held by savers is $\psi_t^S = (1-\lambda)^{-1}$, and so the budget constraint can be expressed as

$$C_t^S = W_tL_t^S + \frac{1}{1-\lambda}D_t + \mathcal{T}_t^S = Y_t^s.$$

As usual, households maximise utility, where agents possess preferences as in (6). So, as before, $\sigma = -u_{CC}^iC^i/u_C^i$; but additionally note that $\varphi = u_{LL}^iL^i/u_L^i$ is the inverse-Frisch elasticity of labour supply, and we have the standard **intratemporal** Euler equation,

$$u_L^i(L_t^i) = W_tu_C^i(C_t^i).$$

Log-linearising about the deterministic steady state gives the following labour supply condition for household i:

$$\varphi\hat{L}_t^i = \hat{W}_t - \sigma\hat{C}_t^i.$$

Since both households share the same elasticities, we can aggregate to get

$$\varphi\hat{L}_t = \hat{W}_t - \sigma\hat{C}_t,$$

and with $\hat{C}_t = \hat{Y}_t = \hat{L}_t$ we can write

$$\hat{W}_t = \left(\sigma + \varphi\right)\hat{C}_t. \tag{12}$$

Also, as before, the consumption Euler equation is

$$\hat{C}_t^S = \mathbb{E}_t\hat{C}_{t+1}^S - \sigma^{-1}\hat{r}_t, \tag{13}$$

noting that the H households do not have an Euler equation.

Typically in TANK models, we require the H household budget constraint as an equilibrium condition. But before doing so, we need to specify rules for fiscal transfers (to be done soon). But before that, we will cover the supply side of the economy.

3. Bilbiie notes that unlike in, say, Galí, López-Salido, and Vallés (2007), households here do not trade their assets. They instead hold and price the assets. This highlights the role of income inequality and profits.

Firms. Production follows a standard NK textbook setup (Galí, 2015). Households consume a basket of differentiated goods indexed by $j \in [0, 1]$,

$$C_t = \left(\int_0^1 C_t(j)^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)},$$

where ϵ is the elasticity of substitution. Demand for good j is

$$C_t(j) = \left[\frac{P_t(j)}{P_t} \right]^{-\epsilon} C_t,$$

where $P_t(j)/P_t$ is good j's relative price and P_t is the aggregate price index,

$$P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} dj.$$

Monopolistically competitive firms produce each type of good with the following linear production technology:

$$Y_t(j) = L_t(j),$$

with real marginal cost W_t.

Profits are given by

$$D_t(j) = (1 + \tau^S) \left[\frac{P_t(j)}{P_t} \right] Y_t(j) - W_t L_t(j) - T_t^F.$$

Assume that the government addresses distortions and inefficiencies arising my market distortions by implementing a production subsidy so that the desired mark up is:

$$\frac{P_t(j)^*}{P_t} = 1 = \frac{\epsilon W_t^*}{(1 + \tau^S)(\epsilon - 1)},$$

where the optimal subsidy to achieve this is $\tau^S = (\epsilon - 1)^{-1}$. This subsidy is financed by a tax on firms:

$$\mathfrak{T}_t^F = \tau^S Y_t.$$

Therefore, total profits are given by

$$D_t = Y_t - W_t L_t.$$

A common convention in TANK models is the assumption of full insurance – or equivalent steady state consumption between the different households. In other words, the taxes on firms leads to zero profits in steady state ($D = 0$) and $C^H = C^S = C$. Log-linearising about the steady state, where $\hat{D}_t = \ln(D_t/Y)$, implies that profits vary inversely with the real wage,

$$\hat{D}_t = -\hat{W}_t. \tag{14}$$

This is an income effect for S-households – more on this later.

Fiscal policy. The government adopts a redistributive tax system. It taxes profits at rate τ^D and rebates these taxes to H households in the form of lump-sum transfers,

$$\mathcal{T}_t^H = \frac{\tau^D}{\lambda} D_t.$$

It turns out, that this is a key transmission channel for monetary policy.

Market clearing. Goods and labour markets clear so that

$$Y_t = C_t \equiv \lambda C_t^H + (1-\lambda) C_t^S,$$

and

$$L_t \equiv \lambda L_t^H + (1-\lambda) L_t^S,$$

whereby $L^H = L^S = L$. These market clearing conditions can be written as

$$\hat{Y}_t = \hat{C}_t = \lambda \hat{C}_t^H + (1-\lambda) \hat{C}_t^S, \tag{15}$$

and

$$\hat{L}_t = \lambda \hat{L}_t^H + (1-\lambda) \hat{L}_t^S. \tag{16}$$

Finally, we have everything we need to also add in the log-linearised budget constraint of the H household:

$$\hat{C}_t^H = \hat{W}_t + \hat{L}_t^H + \frac{\tau^D}{\lambda} \hat{D}_t. \tag{17}$$

Additionally, if prices are sticky we can add in an NKPC and Taylor rule (Fisher equation).

Now, the key point to capture the effects of inequality is to express individual variables as functions of aggregates. Start with the log-linearised H household budget constraint and substitute in (14) and $\hat{L}_t^i = \varphi^{-1} \hat{W}_t - \frac{\sigma}{\varphi} \hat{C}_t^i$ to write:

$$\left(1 + \frac{\sigma}{\varphi}\right) \hat{C}_t^H = \left(1 + \frac{1}{\varphi} - \frac{\tau^D}{\lambda}\right) \hat{W}_t.$$

Then use the aggregate labour supply condition (12) and the fact that $\hat{Y}_t = \hat{L}_t = \hat{C}_t$ to simply write

$$\hat{C}_t^H = \chi \hat{C}_t,$$

and recall that since H households consume all their income, we can also write

$$\hat{C}_t^H = \hat{Y}_t^H = \chi \hat{Y}_t, \tag{18}$$

where

$$\chi = \frac{\left(1 - \frac{\tau^D}{\lambda} + \frac{1}{\varphi}\right)(\varphi + \sigma)}{1 + \frac{\sigma}{\varphi}} = 1 + \varphi \left(1 - \frac{\tau^D}{\lambda}\right).$$

What is (18) telling us? Despite the hand-to-mouth households consuming all their income, their consumption and income co-moves either by less or more than one-to-one with aggregate income or consumption. In other words, χ is the elasticity of H-type households' consumption with respect to aggregate income – or the cyclicality. In the baseline TANK model, χ depends mainly on fiscal redistribution and labour supply. This will pin down the amplification of monetary (and fiscal) policy and shocks. Note that as Bilbiie states, $\chi = 1$ (by either infinitely elastic labour, $\varphi = 0$, or neutral redistribution, $\tau^D = \lambda$), nests the "Campbell-Mankiw benchmark" case.

What about S-type household income? Assuming that taxes on profits are an automatic stabiliser,[4] define

$$\mathcal{T}_t^S = -\frac{\tau^D}{1-\lambda} D_t.$$

Use this in the S-household budget constraint to then write an expression for their income,

$$\hat{Y}_t^S = \hat{W}_t + \hat{L}_t^S + \frac{1-\tau^D}{1-\lambda} \hat{D}_t.$$

Then do some algebra by using the H-type and aggregate labour supply curves, profits (14), and $\hat{C}_t = \hat{Y}_t = \hat{L}_t$ to write

$$\hat{C}_t^S = \frac{1-\lambda\chi}{1-\lambda} \hat{Y}_t. \tag{19}$$

Mechanically, we also have that if H-household's elasticity to aggregate income is greater (less) than one, then the S-household's elasticity, $(1-\lambda\chi)/(1-\lambda)$, must be less (greater) than one. As a bonus, we can substitute (19) into (13) to write

$$\hat{C}_t = \mathbb{E}_t \hat{C}_{t+1} - \frac{1-\lambda}{\sigma(1-\lambda\chi)} \hat{r}_t. \tag{20}$$

Finally, define cyclical income inequality, γ_t, as simply

$$\gamma_t \equiv \hat{Y}_t^S - \hat{Y}_t^H = \frac{1-\chi}{1-\lambda} \hat{Y}_t.$$

By construction, $\chi < 1$ ($\chi > 1$) coincides with procyclical (countercyclical) income inequality.

TANK Aggregate Demand. Under RANK, when aggregate demand increases, the real wage increases (since prices are fixed in the short-run). But this lowers profits (since wages are equal to the marginal cost). Since there is only the representative household, the gain from real wages and the loss in economic rents cancel out. TANK breaks this neutrality result because there is an externality imposed by hand-to-mouth households on saver households through an income effect.

Start with the case of no redistribution ($\tau^D = 0$):

- Suppose demand $\uparrow$ and so $\hat{W} \uparrow$.

4. In other words,

$$\lambda \mathcal{T}_t^H = \tau^D D_t = -(1-\lambda)\mathcal{T}_t^S.$$

- $\hat{Y}^H \uparrow$ and $\hat{C}^H$ since H-households do not have a negative income effect from a rise in wages.
- This leads to another round of an increase in aggregate demand, which shifts labour demand further again, and so on and so forth...
- The result is that S-households end up paying for this because they end up working more due to incurring negative income effect costs due to the rise in the marginal cost.

But $\tau^D > 0$ dampens this channel as $\tau^D \to \chi \downarrow$. H-households internalise some of the negative income effect of higher marginal costs, and so demand does not increase as much as the no redistribution scenario.

The Cambpell-Mankiw benchmark ($\chi = 1$) occurs when the distribution of profits is uniform and so there is no income effect, $\tau^D = \lambda$, or when agents are perfectly insured through wages, $\varphi = 0$. When $\tau^D > 1$, H-households receive a disproportionate share of initial profits – the expansion in aggregate demand is smaller than the initial impulse as the H-households anticipate a fall in their income ($\chi < 1$) and S-households work less and keep the higher profits.

To get the aggregate consumption Euler, recall that for S-households, their problem is identical to the representative household in RANK. So take (10), replace i with S, and use $\hat{Y}_t^S = \hat{W}_t + \hat{L}_t^S + \frac{1-\tau^D}{1-\lambda}\hat{D}_t$, $\hat{D}_t = -\hat{W}_t$, and $\hat{L}_t^S = \hat{L}_t = \hat{Y}_t = \hat{C}_t$ to write the aggregate PE curve:

$$\hat{C}_t = [1 - \beta(1 - \lambda\chi)]\,\hat{Y}_t - \frac{(1-\lambda)\beta}{\sigma}\hat{r}_t + \beta(1-\lambda\chi)\mathbb{E}_t\hat{C}_{t+1}, \tag{21}$$

which generalises to Cambpell and Mankiw's equation with arbitrary $\chi \neq 1$.

As we did in the RANK case, first get the Keynesian multiplier Ω for a real interest rate cut by assuming $\mathbb{E}_t\hat{C}_{t+1} = \rho\hat{C}_t$, and use the aggregate Euler-IS curve, (20):

$$\Omega \equiv \frac{d\hat{C}_t}{d(-\hat{r}_t)} = \frac{1-\lambda}{\sigma(1-\rho)(1-\lambda\chi)}.$$

Then get the direct effect of the interest rate reduction, Ω_D, by using the aggregate PE curve (21) and keeping income fixed:

$$\Omega_D \equiv \left.\frac{d\hat{C}_t}{d(-\hat{r}_t)}\right|_{\hat{Y}_t=\bar{Y}} = \frac{\beta(1-\lambda)}{\sigma\left[1-\beta\rho(1-\lambda\chi)\right]},$$

which then gives the indirect effect,

$$\Omega_I \equiv \Omega - \Omega_D = \frac{(1-\beta(1-\lambda\chi))(1-\lambda)}{\sigma(1-\rho)(1-\lambda\chi)(1-\beta\rho(1-\lambda\chi))}.$$

We can then impute the aggregate MPC as

$$\omega \equiv \frac{\Omega_I}{\Omega} = \frac{1-\beta(1-\lambda\chi)}{1-\beta\rho(1-\lambda\chi)}.$$

This leads us to Bilbiie's second proposition:

Proposition (Bilbiie (2020) Proposition 2)**.** *In TANK, the aggregate MPC ω and multiplier Ω (for an interest rate cut of persistence ρ) are:*

$$\omega = \frac{1-\beta(1-\lambda\chi)}{1-\beta\rho(1-\lambda\chi)};\ \Omega = \frac{1-\lambda}{\sigma(1-\rho)(1-\lambda\chi)}.$$

There is amplification ($d\Omega/d\lambda > 0$) if and only if income inequality is countercyclical:

$$\chi > 1. \tag{22}$$

To understand the proposition, assume that shocks to $\hat{r}_t$ are transitory ($\rho = 0$), and recall the issues with RANK:

- General equilibrium effect of $\hat{r}_t \downarrow$: $\Omega = \sigma^{-1}$, which is close to zero empirically.
- MPC is $\omega = 1-\beta$, which is also close to zero.

With TANK, the general equilibrium effect of a rate cut is now $\Omega = (1-\lambda)/[\sigma(1-\lambda\chi)]$ and the MPC becomes $1-\beta(1-\lambda\chi)$. The existence of hand-to-mouth households reduces the direct effect, Ω_D, of interest rate changes because H-households are basically not directly affected by them – the saver households are the ones that engage in intertemporal substitution since only they have a consumption Euler equation. So while the direct effect is proportionally lower in TANK, the indirect effect is stronger due to the higher MPC. This is because H-households have an MPC of unity, and thus $\partial\omega/\partial\lambda = \beta\chi$, where recall that λ was the population ratio of hand-to-mouth households. In other words, this is effect of λ on the slope of the PE curve.

Verifying amplification in (22) is straightforward:

$$\frac{\partial\Omega}{\partial\lambda} = \frac{(\chi-1)\Omega^{\text{RANK}}}{(1-\lambda\chi)^2}.$$

Amplification (over RANK) occurs when the shift effect of the PE curve relative to RANK,

$$\frac{\partial \frac{\Omega_D}{\Omega_D^{\text{RANK}}}}{\partial\lambda} = -\beta,$$

dominates the aforementioned slope effect, i.e., $-\beta + \beta\chi > 0$. When this is not the case, then there is dampening. Dampening occurs when inequality is procyclical ($\chi < 1$) as the hand-to-mouth households internalise the negative effect of a potential wage increase on their income (via transfers). So rather than increase their demand after a labour demand increase, they decrease it. This mechanism is covered in the recent quantitative HANK literature as the "earnings channel" (Auclert, 2019).

Additionally, amplification occurs so long as $\lambda < \chi^{-1}$. If $\lambda > \chi^{-1}$ then an expansion to a real interest rate decrease can no longer be an equilibrium. The income effect on S-households starts dominating the IS curve swivels – this is Bilbiie (2008)'s "inverted AD curve" logic.

Fiscal Multipliers in TANK. Given that TANK models usually comprise of "constrained" and "unconstrained" agents, the macroeconomics literature focused on fiscal policy and multipliers. Here we use the NK cross to analyse these issues. Start with the baseline case that government engages in wasteful spending, G_t, funded by lump-sum taxes $\mathcal{T}_t$, of which each agent pays $\mathcal{T}_t^i$.

Exogenous redistribution: assume that H-households pay an arbitrary share of total taxes, $\lambda\mathcal{T}_t^H = \alpha\mathcal{T}_t$, while S-households pay $(1-\lambda)\mathcal{T}_t^S = (1-\alpha)\mathcal{T}_t$. Here α can be interpreted as a proxy parameter for tax regressivity. Assume that steady state values for G_t and $\mathcal{T}_t$ are zero for simplicity (with $\hat{\mathcal{T}}_t^H = \mathcal{T}_t^H/Y$ and $\hat{G}_t = G_t/Y$) then decompose $\hat{\mathcal{T}}_t^H$ as

$$\hat{\mathcal{T}}_t^H = \frac{\alpha}{\lambda}\hat{\mathcal{T}}_t = \frac{\alpha}{\lambda}\hat{G}_t = \hat{G}_t - \left(1 - \frac{\alpha}{\lambda}\right)\hat{\mathcal{T}}_t, \tag{23}$$

the sum of a tax (spending) increase and a transfer to the hand-to-mouth households whenever $\alpha < \lambda$. If $\alpha > \lambda$ then there is a transfer from the H-households. These are different compared to the endogenous redistribution mechanism we observed in the χ term (τ^D/λ).

With $\hat{y}_t^i = \hat{Y}_t^i - \hat{\mathcal{T}}_t^i$, (23) implies

$$\hat{C}_t^H = \hat{y}_t^H = \chi\hat{y}_t + \zeta\left(\chi - \frac{\alpha}{\lambda}\right)\hat{G}_t,$$

with χ defined as before and

$$\zeta = \frac{1}{1+\frac{\sigma}{\varphi}}.$$

We then get

$$\hat{Y}_t = \hat{C}_t + \hat{G}_t,$$
$$\hat{C}_t^H = \hat{W}_t + \hat{L}_t^H + \frac{\tau^D}{\lambda}\hat{D}_t - \hat{\mathcal{T}}_t^H,$$

and replace L_t^H in the above equation using $\varphi L_t^i = \hat{W}_t - \sigma\hat{C}_t^i$ and $\hat{D}_t = -\hat{W}_t$ to get

$$\left(1 + \frac{\sigma}{\varphi}\right)\hat{C}_t^H = \left(1 - \frac{\tau^D}{\lambda} + \frac{1}{\varphi}\right)\hat{W}_t - \hat{\mathcal{T}}_t^H,$$

then use the aggregate labour supply condition and some other aggregate identities, $\hat{Y}_t = \hat{L}_t = \hat{C}_t + \hat{G}_t \implies \hat{W}_t = (\varphi+\sigma)\hat{C}_t + \varphi\hat{G}_t$, to write the above as

$$\hat{C}_t^H = \chi\hat{C}_t + \zeta\left(\chi\hat{G}_t - \hat{\mathcal{T}}_t^H\right). \tag{24}$$

The second term captures the impact of fiscal variables on H-households, the coefficient ζ is the elasticity of the H-household's consumption to a transfer and governs the strength of the income effect relative to substitution. This term is zero when labour supply is infinitely elastic ($\varphi = 0$), equal to unity when labour is inelastic or when the income effect (σ) is zero (the case of log preferences).

Much like with the method of getting (21), one can get the aggregate PE curve with fiscal policy:

$$\hat{C}_t = [1 - \beta(1 - \lambda\chi)]\,\hat{y}_t - \frac{(1-\lambda)\beta}{\sigma}\hat{r}_t + \beta(1-\lambda\chi)\mathbb{E}_t\hat{C}_{t+1} + \beta\lambda\zeta\left(\chi - \frac{\alpha}{\lambda}\right)(\hat{G}_t - \mathbb{E}_t\hat{G}_{t+1}), \tag{25}$$

and with $\hat{C}_t = \hat{y}_t$ the aggregate Euler-IS curve is

$$\hat{C}_t = \mathbb{E}_t\hat{C}_{t+1} - \frac{1-\lambda}{\sigma(1-\lambda\chi)}\hat{r}_t + \frac{\lambda\zeta}{1-\lambda\chi}\left(\chi - \frac{\alpha}{\lambda}\right)(\hat{G}_t - \mathbb{E}_t\hat{G}_{t+1}). \tag{26}$$

With a bit of substitution (for $\hat{y}_t = \hat{Y}_t - \hat{\mathcal{T}}_t$), we get the fiscal multiplier,

$$\mathcal{M} \equiv \frac{d\hat{Y}_t}{d\hat{G}_t} = 1 + \frac{\lambda\zeta}{1-\lambda\chi}\left(\chi - \frac{\alpha}{\lambda}\right). \tag{27}$$

In TANK $\mathcal{M} > 1$ – as opposed to $\mathcal{M} = 1$ in RANK, if condition (22) holds generally: $\chi > \alpha/\lambda$, where the RHS is the share of taxes that H-households need to pay.

- If they pay nothing ($\alpha = 0$), then $\mathcal{M} > 1$ for any value $\chi > 0$.
- If tax is uniform ($\alpha = \lambda$) then $\mathcal{M} > 1$ if and only if $\chi > 1$ just like in (22).
- The RANK condition (no multiplier) holds if $\lambda = 0$, or $\varphi = 0$ (infinitely elastic labour), or when $\chi = \alpha/\lambda$.

The literature on fiscal multipliers in New Keynesian models is vast. The classic references are: Galí, López-Salido, and Vallés (2007) which numerically analysed spending multipliers in a TANK model with capital; and Bilbiie, Meier, and Müller (2008) and Monacelli and Perotti (2011) which looked at spending multipliers analytically; and Bilbiie, Monacelli, and Perotti (2013) focused on resdistribution or transfer multipliers in a TANK model with financial market imperfections. Then there are papers such as Eggertsson and Krugman (2012) which assessed government spending multipliers at the effective lower bound. Then, more recently, Ghassibe and Zanetti (2022) build a model framework to analyse the state dependence of fiscal multipliers.

2 A Simple HANK Model

One big omission in TANK (and RANK, obviously): precautionary savings. Here we derive a tractable HANK (THANK) model where by agents can be in either a hand-to-mouth (H) state or an unconstrained saver (S) state. Unlike the TANK case, here you can think of individual agents switching exogenously between the two states. Agents have access to insurance – full within type (after idiosyncratic uncertainty revealed) and limited across types. There are two assets: bonds which are liquid, and stocks which are illiquid. Finally, bonds cannot be traded (no equilibrium liquidity) for simplification purposes.

The exogenous change of state follows a Markov chain. The probability to stay as type S is $p(S|S) = s$, the probability to stay as type H is $p(H|H) = h$, the probability to transition from type H to type S is

$p(S|H) = 1 - h$, and the probability to transition from type S to type H is $p(H|S) = 1 - s$. The mass of H households follows a stationary distribution,

$$\lambda = \frac{1-s}{2-s-h},$$

with the requirement that $s \geq 1 - h$, and is the solution to

$$\begin{pmatrix} \lambda & 1-\lambda \end{pmatrix} \begin{pmatrix} h & 1-h \\ 1-s & s \end{pmatrix} = \begin{pmatrix} \lambda & 1-\lambda \end{pmatrix}.$$

The equilibrium is characterised as follows. In every period, individuals that are H wish to borrow but are unable to due to borrowing constraints. Neither can they access equity portfolios (since these are held by savers). Like TANK, we assume that all H-households consume all their income, $C_t^H = Y_t^H$. Since the transition probabilities are independent of history, and there is perfect insurance amongst types, all agents that hand-to-mouth in a given period have the same income and consumption. Conversely, savers save in order to self-insure against the risk of becoming hand-to-mouth. But since equity is costly to liquidate, they can engage in precautionary savings through bonds. But since H-individuals cannot borrow, and there is no government-provided liquidity, bonds are in zero net supply (Krusell, Mukoyama, and Smith, 2011). As in RANK, an Euler equation prices these bonds even though they are not traded.

But the Euler equation takes factors in the transition probabilities to the constrained H state unlike in TANK. In line with some HANK models, such as Kaplan, Moll, and Violante (2018), there is a [crude] distinction between liquid (bonds) and illiquid (equity) assets: in equilibrium, there is infrequent (limited) participation in the stock market.

For derivation, started by noting the following per-capita bond flows. Let B_t^S be the beginning of period t after consumption-saving choice – also after changing state and pooling. Let A_t^S be the end of period $t-1$ after consumption-saving choice but before moving:

$$\begin{aligned} (1-\lambda)B_t^S &= (1-\lambda)sA_t^S + (1-\lambda)(1-s)A_t^H, \\ \lambda B_t^H &= (1-\lambda)(1-s)A_t^S + \lambda h A_t^H. \end{aligned}$$

Rescale and use $\lambda = \frac{1-s}{1-s+1-h}$:

$$\begin{aligned} B_t^S &= sZ_t^S + (1-s)A_t^H, \\ B_t^H &= (1-\lambda)A_t^S + hA_t^H. \end{aligned}$$

The family (head) optimisation problem is:

$$\mathbb{V}(B_{t-1}^S, B_{t-1}^H, \psi_{t-1}) = \max_{\{C_t^S, A_t^S, A_t^H, C_t^H, \psi_t\}} (1-\lambda)u(C_t^S) + \lambda u(C_t^H) + \beta \mathbb{E}_t \mathbb{V}(B_t^S, B_t^H, \psi_t),$$

subject to:

$$
\begin{aligned}
C_t^S + A_t^S + V_t\psi_t &= Y_t^S + r_{t-1}B_{t-1}^S + \psi_{t-1}(V_t + D_t), \\
C_t^H + A_t^H &= Y_t^H + r_{t-1}B_{t-1}^H, \\
A_t^S, A_t^H &\geq 0,
\end{aligned}
$$

and the laws of motion for bond flows relating the A^i to the B^i.

Euler equations. The FOCs are the following:

$$
u'(C_t^S) \geq \beta\mathbb{E}_t\left[\frac{V_{t+1} + D_{t+1}}{V_t}u'(C_{t+1}^S)\right], \quad \psi_t = \psi_{t-1} = (1-\lambda)^{-1}, \tag{28}
$$

which is an Euler equation corresponding to the choice of equity. There is no self-insurance motive, as equity cannot be carried when an individual transitions to the H island – this equation is the same as with a representative agent. The FOC for the bond choice of S-island agents is given by

$$
\begin{aligned}
u'(C_t^S) &\geq \beta\mathbb{E}_t\left[r_t\left\{su'(C_{t+1}^S) + (1-s)u'(C_{t+1}^H)\right\}\right], \\
0 &= A_t^S\left\{u'(C_t^S) - \beta\mathbb{E}_t\left[r_t\left\{su'(C_{t+1}^S) + (1-s)u'(C_{t+1}^H)\right\}\right]\right\},
\end{aligned} \tag{29}
$$

where the second equation is a slackness condition. The FOC for the bond choice of agents in the H-island is:

$$
\begin{aligned}
u'(C_t^H) &\geq \beta\mathbb{E}_t\left[r_t\left\{(1-h)u'(C_{t+1}^S) + hu'(C_{t+1}^H)\right\}\right], \\
0 &= A_t^H\left\{u'(C_t^H) - \beta\mathbb{E}_t\left[r_t\left\{(1-h)u'(C_{t+1}^S) + hu'(C_{t+1}^H)\right\}\right]\right\}.
\end{aligned} \tag{30}
$$

Recall that only bonds can be used by agents as they transition across H and S islands. With this market structure, Euler equations (29) and (30) are similar as if they were in a Bewley-Huggett-Aiyagari economy with incomplete markets. Here, $1 - s$ is the uninsurable risk to switch to a "bad" state next period, for which only bonds can be used to self-insure. This is the precautionary savings motive in this model.

Then, make two more additional assumptions for analytical tractability: assume that the budget constraint for H-island individuals bind with equality and that the Euler equation (30) is a strict inequality (and so $A_t^H = 0$):

$$
C_t^H = Y_t^H.
$$

The motivation for this could be due to, say, a liquidity shock or an impatience shock which makes them want to consume more today. Second, assume that we work with the zero-liquidity limit. So that even though the demand for bonds from S is well defined ($A_t^S > 0$), the supply of bonds is zero so trade does not occur in equilibrium. Thus, the Euler equation (29) binds with equality.

So, the only equation that differs from TANK is the "self-insurance" Euler equation against the risk of moving to the H-island:

$$(C_t^S)^{-\sigma} = \beta \mathbb{E}_t \left[r_t \left\{ s(C_{t+1}^S)^{-\sigma} + (1-s)(C_{t+1}^H)^{-\sigma} \right\} \right], \tag{31}$$

which comes from (29) and standard CRRA preferences. Log-linearising this equation about the symmetric steady (and with $D = 0$) state yields

$$\hat{C}_t^S = s\mathbb{E}_t \hat{C}_{t+1}^S + (1-s)\mathbb{E}_t \hat{C}_{t+1}^H - \sigma^{-1}\hat{r}_t. \tag{32}$$

Recall that like the TANK case we have (18), regardless of the redistribution scheme that determines χ:

$$\hat{C}_t^H = \chi \hat{Y}_t,$$
$$\hat{C}_t^S = \frac{1-\lambda\chi}{1-\lambda}\hat{Y}_t.$$

Substituting these into (32):

$$\begin{aligned}
\hat{Y}_t &= s\mathbb{E}_t\hat{Y}_{t+1} + \frac{(1-s)(1-\lambda)\chi}{1-\lambda\chi}\mathbb{E}_t\hat{Y}_{t+1} - \frac{1-\lambda}{\sigma(1-\lambda\chi)}\hat{r}_t \\
&= \left[\frac{s(1-\lambda\chi) + (1-s)(1-\lambda)\chi}{1-\lambda\chi}\right]\mathbb{E}_t\hat{Y}_{t+1} - \frac{1-\lambda}{\sigma(1-\lambda\chi)}\hat{r}_t \\
&= \left[\frac{s - s\lambda\chi + \chi - \lambda\chi - s\chi + s\lambda\chi + 1 - 1 + \lambda\chi - \lambda\chi}{1-\lambda\chi}\right]\mathbb{E}_t\hat{Y}_{t+1} - \frac{1-\lambda}{\sigma(1-\lambda\chi)}\hat{r}_t \\
&= \underbrace{\left[1 + \frac{(\chi-1)(1-s)}{1-\lambda\chi}\right]}_{\text{HANK}}\mathbb{E}_t\hat{Y}_{t+1} - \frac{1}{\sigma}\underbrace{\frac{1-\lambda}{(1-\lambda\chi)}}_{\text{TANK}}\hat{r}_t,
\end{aligned}$$

and get the Euler-IS equation for the HANK model, which leads us to Bibliie's next proposition:

Proposition (Bilbiie (2020) Proposition 3). *In the analytical HANK model, the aggregate Euler-IS curve is:*

$$\hat{C}_t = \delta\mathbb{E}_t\hat{C}_{t+1} - \frac{1-\lambda}{\sigma(1-\lambda\chi)}\hat{r}_t, \quad \delta = 1 + \frac{(\chi-1)(1-s)}{1-\lambda\chi}, \tag{33}$$

with idiosyncratic uncertainty $s < 1$, this is characterised by

- ***discounting**: $\delta < 1$ if and only if $\chi < 1$ (procyclical inequality); and*
- ***compounding**: $\delta > 1$ if and only if $\chi > 1$ (countercyclical inequality).*

Assuming that $\lambda < \chi^{-1}$; otherwise, the elasticity of aggregate demand to interest rates changes sign when $\chi > 1$ with complicated implications for δ.

Intuition is as follows. Consider the RANK case: Good news about future income implies a one-to-one increase in aggregate demand today as the household wants to substitute consumption towards the present. Without any assets (or capital), income adjust to deliver this as markets clear. The same occurs in TANK with permanent idiosyncratic shocks ($s = h = 1$), $\delta = 1$.

When there is "discounting", $\delta < 1$, which generalises McKay, Nakamura, and Steinsson (2016) (nested for $\chi = 0 \implies \delta = s = 1 - h = 1 - \lambda$). When households receive good news about future **aggregate** income or consumption, they also realise that in some periods in the future they will be constrained and not benefit fully from it – recall that χ denoted the H-household's elasticity consumption/income to aggregate income. Agents self-insure or engage in precautionary savings against this and so their consumption today is less than that in a RANK or economy without uncertainty. As there is no asset to save in (or capital), the reduction in consumption and consequent increase in saving results in a drop in output. Intuitively, precautionary saving is strengthened the higher the risk of idiosyncratic uncertainty $(1-s)$, the lower is χ, and the longer an agent expects to be in the H-state (higher λ at a given s) implies higher h).

The easiest way to think about the "compounding" case is that H-agents are induced to increase their consumption both contemporaneously but also intertemporally when they receive good news about future income. Because of progressive redistribution, they demand less saving, and this effect is magnified with higher risk $(1 - s)$, χ, and λ.

One final aside, we can also nest the case of acyclical risk where $s = 0$ (Woodford, 1990) and $\lambda = 0.5$ so we have

$$\hat{C}_t = \frac{\chi}{2-\chi}\mathbb{E}_t\hat{C}_{t+1} - \frac{1}{\sigma(2-\chi)}\hat{r}_t,$$

where $\delta \equiv \chi/(2-\chi) \gtreqless 1$ if and only if $\chi \gtreqless 1$.

NK Cross with HANK. To derive the aggregate PE curve, start with (32),

$$\hat{C}_t^S = s\mathbb{E}_t\hat{C}_{t+1}^S + (1-s)\mathbb{E}_t\hat{C}_{t+1}^H - \sigma^{-1}\hat{r}_t,$$

and iterate forward:

$$\mathbb{E}_t\hat{C}_{t+1}^S = s\mathbb{E}_{t+1}\hat{C}_{t+2}^S + (1-s)\mathbb{E}_{t+1}\hat{C}_{t+2}^H - \sigma^{-1}\mathbb{E}_t\hat{r}_{t+1}.$$

Then substitute the above back into (32) to get

$$\begin{aligned}
\hat{C}_t^S &= s\mathbb{E}_t\left[s\mathbb{E}_{t+1}\hat{C}_{t+2}^S + (1-s)\mathbb{E}_{t+1}\hat{C}_{t+2}^H - \sigma^{-1}\mathbb{E}_t\hat{r}_{t+1}\right] + (1-s)\mathbb{E}_t\hat{C}_{t+1}^H - \sigma^{-1}\hat{r}_t \\
&= s^2\mathbb{E}_t\hat{C}_{t+2}^S + s(1-s)\mathbb{E}_t\hat{C}_{t+2}^H + (1-s)\mathbb{E}_t\hat{C}_{t+1}^H - \sigma^{-1}\left(\hat{r}_t + s\mathbb{E}_t\hat{r}_{t+1}\right) \\
\Leftrightarrow \hat{C}_t^S &= s^i\mathbb{E}_t\hat{C}_{t+i}^S + \mathbb{E}_t\sum_{k=0}^{i-1}s^k\left[(1-s)\hat{C}_{t+1+k}^H - \sigma^{-1}\hat{r}_{t+k}\right]
\end{aligned}$$

Additionally, we can use the SDF instead of the real interest rate to write (32) as

$$\hat{C}_t^S = s\mathbb{E}_t\hat{C}_{t+1}^S + (1-s)\mathbb{E}_t\hat{C}_{t+1}^H + \sigma^{-1}\mathbb{E}_t\hat{\Lambda}_{t,t+1}^S. \tag{34}$$

Then do the same forward iteration and substitution trick as above:

$$\begin{aligned}\hat{C}_t^S &= s\mathbb{E}_t\left[s\mathbb{E}_{t+1}\hat{C}_{t+2}^S + (1-s)\mathbb{E}_{t+1}\hat{C}_{t+2}^H + \sigma^{-1}\mathbb{E}_t\hat{\Lambda}_{t,t+2}^S\right] + (1-s)\mathbb{E}_t\hat{C}_{t+1}^H + \sigma^{-1}\mathbb{E}_t\hat{\Lambda}_{t,t+1} \\ &= s^2\mathbb{E}_t\hat{C}_{t+2}^S + s(1-s)\mathbb{E}_t\hat{C}_{t+2}^H + (1-s)\mathbb{E}_t\hat{C}_{t+1}^H + \sigma^{-1}\mathbb{E}_t\left[\hat{\Lambda}_{t,t+1}^S + s\hat{\Lambda}_{t,t+2}\right] \\ \Leftrightarrow \hat{C}_t^S &= s^i\mathbb{E}_t\hat{C}_{t+i}^S + \mathbb{E}_t\sum_{k=0}^{i-1}s^k\left[(1-s)\hat{C}_{t+1+k}^H + \sigma^{-1}\hat{\Lambda}_{t,t+k}^S\right].\end{aligned}$$

But also recall that $\hat{\Lambda}_{t,t+i} = \hat{\Lambda}_{t,t+1} + \hat{\Lambda}_{t+1,t+2} + ... + \hat{\Lambda}_{t+i-1,t+i}$, and so (34) can be written as

$$\begin{aligned}\sigma^{-1}\mathbb{E}_t\hat{\Lambda}_{t,t+i}^S &= \hat{C}_t^S - s\mathbb{E}_t\hat{C}_{t+1}^S - (1-s)\mathbb{E}_t\hat{C}_{t+1}^H + \mathbb{E}_t\hat{C}_{t+1}^S - s\mathbb{E}_t\hat{C}_{t+2}^S - (1-s)\mathbb{E}_t\hat{C}_{t+2}^H + \\ &\quad \cdots + \mathbb{E}_t\hat{C}_{t+i-1}^S - s\mathbb{E}_t\hat{C}_{t+i}^S - (1-s)\mathbb{E}_t\hat{C}_{t+i}^H \\ \therefore \sigma^{-1}\mathbb{E}_t\hat{\Lambda}_{t,t+i}^S + \mathbb{E}_t\hat{C}_{t+i}^S &= \hat{C}_t^S + (1-s)\mathbb{E}_t\sum_{k=1}^{i}\left(\hat{C}_{t+k}^S - \hat{C}_{t+k}^H\right).\end{aligned}$$

Then as we did with (3) and (5), log-linearise the budget constraint:

$$\mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left(\hat{\Lambda}_{t,t+i}^S + \hat{C}_{t+i}^S\right) = \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left(\hat{\Lambda}_{t,t+i}^S + \hat{Y}_{t+i}^S\right).$$

Now, do some algebra, using the above results:

$$\begin{aligned}\mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left[\hat{\Lambda}_{t,t+i}^S + \hat{C}_{t+i}^S + \left(\frac{1}{\sigma}-1\right)\hat{\Lambda}_{t,t+i}^S\right] &= \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left[\hat{\Lambda}_{t,t+i}^S + \hat{Y}_{t+i}^S + \left(\frac{1}{\sigma}-1\right)\hat{\Lambda}_{t,t+i}^S\right] \\ \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left(\frac{1}{\sigma}\hat{\Lambda}_{t,t+i}^S + \hat{C}_{t+i}^S\right) &= \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left(\frac{1}{\sigma}\hat{\Lambda}_{t,t+i}^S + \hat{Y}_{t+i}^S\right) \\ \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left[\hat{C}_t^S + (1-s)\mathbb{E}_t\sum_{k=1}^{i}\left(\hat{C}_{t+k}^S - \hat{C}_{t+k}^H\right)\right] &= \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left(\frac{1}{\sigma}\hat{\Lambda}_{t,t+i}^S + \hat{Y}_{t+i}^S\right) \\ \frac{1}{1-\beta}\hat{C}_t^S + \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i(1-s)\left[\mathbb{E}_t\sum_{k=1}^{i}\left(\hat{C}_{t+k}^S - \hat{C}_{t+k}^H\right)\right] &= \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left(\frac{1}{\sigma}\hat{\Lambda}_{t,t+i}^S + \hat{Y}_{t+i}^S\right) \\ \frac{1}{1-\beta}\hat{C}_t^S + \frac{1-s}{1-\beta}\mathbb{E}_t\sum_{i=1}^{\infty}\beta^i\left(\hat{C}_{t+i}^S - \hat{C}_{t+i}^H\right) &= \mathbb{E}_t\sum_{i=0}^{\infty}\beta^i\left(\frac{1}{\sigma}\hat{\Lambda}_{t,t+i}^S + \hat{Y}_{t+i}^S\right),\end{aligned}$$

and use $\hat{\Lambda}_{t,t+i} = -\sum_{k=0}^{i} \hat{r}_{t+k}$ and $\hat{\Lambda}_{t,t} = 0$,

$$\begin{aligned}
\frac{1}{1-\beta}\hat{C}_t^S &= \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \sigma^{-1} \hat{\Lambda}_{t,t+i}^S + \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{Y}_{t+i}^S - \frac{1-s}{1-\beta}\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \left(\hat{C}_{t+i}^S - \hat{C}_{t+i}^H\right) \\
\hat{C}_t^S &= \frac{1-\beta}{\sigma}\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{\Lambda}_{t,t+i}^S + (1-\beta)\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{Y}_{t+i}^S - (1-s)\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \left(\hat{C}_{t+i}^S - \hat{C}_{t+i}^H\right) \\
&= \frac{1-\beta}{\sigma}\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{\Lambda}_{t,t+i}^S + (1-\beta)\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{Y}_{t+i}^S - (1-s)\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \left(\hat{C}_{t+i}^S - \hat{C}_{t+i}^H\right) \\
&= (1-\beta)\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{Y}_{t+i}^S - \frac{(1-\beta)}{\sigma}\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{r}_{t+i} - (1-s)\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \left(\hat{C}_{t+i}^S - \hat{C}_{t+i}^H\right) \\
&= \frac{(1-\beta)}{\sigma}\left(0 - \mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \sum_{k=0}^{i} \hat{r}_{t+k}\right) + (1-\beta)\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{Y}_{t+i}^S - (1-s)\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \left(\hat{C}_{t+i}^S - \hat{C}_{t+i}^H\right) \\
&= (1-\beta)\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{Y}_{t+i}^S - \frac{\beta}{\sigma}\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{r}_{t+i} - (1-s)\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \left(\hat{C}_{t+i}^S - \hat{C}_{t+i}^H\right) \\
\hat{C}_t^S &= (1-\beta)\hat{Y}_t^S - \frac{\beta}{\sigma}\hat{r}_t + (1-\beta)\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \hat{Y}_{t+i}^S - \frac{\beta}{\sigma}\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \hat{r}_{t+i} - (1-s)\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \left(\hat{C}_{t+i}^S - \hat{C}_{t+i}^H\right).
\end{aligned}$$

By writing the above form in recursive form, we get our consumption function:

$$\begin{aligned}
\hat{C}_t^S &= (1-\beta)\hat{Y}_t^S - \frac{\beta}{\sigma}\hat{r}_t - (1-s)\beta\mathbb{E}_t\left(\hat{C}_{t+1}^S - \hat{C}_{t+1}^H\right) + \beta\mathbb{E}_t\hat{C}_{t+1}^S \\
&= (1-\beta)\hat{Y}_t^S - \frac{\beta}{\sigma}\hat{r}_t - s\beta\mathbb{E}_t\hat{C}_{t+1}^S + \beta(1-s)\mathbb{E}_t\hat{C}_{t+1}^H,
\end{aligned}$$

noting that

$$\begin{aligned}
\mathbb{E}_t\hat{C}_{t+1}^S &= (1-\beta)\hat{Y}_{t+1}^S - \frac{\beta}{\sigma}\hat{r}_{t+1} + (1-\beta)\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \hat{Y}_{t+1+i}^S \\
&\quad - \frac{\beta}{\sigma}\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \hat{r}_{t+1+i} - (1-s)\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \left(\hat{C}_{t+1+i}^S - \hat{C}_{t+1+i}^H\right) \\
\beta\hat{C}_{t+1}^S &= \beta(1-\beta)\hat{Y}_{t+1}^S - \frac{\beta^2}{\sigma}\hat{r}_{t+1} + \beta(1-\beta)\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \hat{Y}_{t+1+i}^S \\
&\quad - \frac{\beta^2}{\sigma}\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \hat{r}_{t+1+i} - \beta(1-s)\mathbb{E}_t \sum_{i=1}^{\infty} \beta^i \left(\hat{C}_{t+1+i}^S - \hat{C}_{t+1+i}^H\right) \\
&= (1-\beta)\mathbb{E}_t \sum_{i=0}^{\infty} \beta^{1+i} \hat{Y}_{t+1+i} - \frac{\beta}{\sigma}\mathbb{E}_t \sum_{i=0}^{\infty} \beta^{1+i} \hat{r}_{t+1+i} - (1-s)\mathbb{E}_t \sum_{i=0}^{\infty} \beta^{1+i} \left(\hat{C}_{t+1+i}^S - \hat{C}_{t+1+i}^H\right).
\end{aligned}$$

Finally, use $\hat{C}_t^H = \hat{Y}_t^H = \chi\hat{Y}_t$, $\hat{C}_t^S = \frac{1-\lambda\chi}{1-\lambda}\hat{Y}_t$, $\hat{Y}_t^S = \hat{W}_t + \hat{L}_t^S + \frac{1-\tau^D}{1-\lambda}\hat{D}_t$, $\hat{D}_t = -\hat{W}_t$, and $\hat{L}_t^S = \hat{L}_t = \hat{Y}_t = \hat{C}_t$ to aggregate the recursive expression of the consumption function (like we did for (21)) to get the HANK

PE curve:

$$\hat{C}_t = [1 - \beta(1-\lambda\chi)]\,\hat{Y}_t - \frac{(1-\lambda)\beta}{\sigma}\hat{r}_t + \beta\delta(1-\lambda\chi)\mathbb{E}_t\hat{C}_{t+1}. \tag{35}$$

As before, get the Keynesian multiplier,

$$\Omega \equiv \frac{d\hat{C}_t}{d(-\hat{r}_t)} = \frac{1-\lambda}{\sigma(1-\delta\rho)(1-\lambda\chi)},$$

the direct effect of the interest rate reduction,

$$\Omega_D \equiv (1-\omega)\Omega \equiv \left.\frac{d\hat{C}_t}{d(-\hat{r}_t)}\right|_{\hat{Y}_t=\bar{Y}} = \frac{(1-\lambda)\beta}{\sigma\,[1-\beta\delta\rho(1-\lambda\chi)]},$$

the indirect effect,

$$\Omega_I \equiv \Omega - \Omega_D = \frac{(1-\lambda)\,[1-\beta(1-\lambda\chi)]}{\sigma(1-\delta\rho)(1-\lambda\chi)\,[1-\beta\delta\rho(1-\lambda\chi)]},$$

and the aggregate MPC as

$$\omega \equiv \frac{\Omega_I}{\Omega} = \frac{1-\beta(1-\lambda\chi)}{1-\delta\beta\rho(1-\lambda\chi)}.$$

Intuitively, HANK differs to TANK through δ, which itself augments persistence by making shocks either more (less) persistent when $\delta > 1$ ($\delta < 1$): shocks that are about the future are either attenuated or amplified. This is intuitive as precautionary savings is inherently about an agent's perception of future states of the world. Below we will see an application of the HANK model.

2.1 Application: The Forward Guidance puzzle

One application that highlights the novelty of the HANK model is the resolution of the Forward Guidance (FG) puzzle. Consider a simple case where the monetary authority announces an interest rate cut in period $t+T$. Start by iterating (35) forward:

$$\begin{aligned}
\hat{C}_t &= \underbrace{[1-\beta(1-\lambda\chi)]}_{A}\,\hat{Y}_t - \underbrace{\frac{(1-\lambda)\beta}{\sigma}}_{B}\,\hat{r}_t + \underbrace{\beta\delta(1-\lambda\chi)}_{D}\,\mathbb{E}_t\hat{C}_{t+1},\\
\hat{C}_{t+1} &= A\hat{Y}_{t+1} - B\hat{r}_{t+1} + D\mathbb{E}_{t+1}\hat{C}_{t+2},\\
\hat{C}_{t+2} &= A\hat{Y}_{t+2} - B\hat{r}_{t+2} + D\mathbb{E}_{t+2}\hat{C}_{t+3},\\
&\vdots\\
\hat{C}_{t+i} &= A\hat{Y}_{t+i} - B\hat{r}_{t+i} + D\mathbb{E}_{t+i}\hat{C}_{t+1+i}.
\end{aligned}$$

Do some recursive substitution to get

$$\hat{C}_t = A\sum_{i=0}^{\infty} D^i\hat{Y}_{t+i} - B\sum_{i=0}^{\infty} D^i\hat{r}_{t+i} + D^i\mathbb{E}_{t+i}\hat{C}_{t+i},$$

or, and by using the fact that $\lim_{i\to\infty} D^i = 0$:

$$\hat{C}_t = [1-\beta(1-\lambda\chi)] \sum_{i=0}^{\infty} [\beta\delta(1-\lambda\chi)]^i \, \hat{Y}_{t+i} - \frac{(1-\lambda)\beta}{\sigma} \sum_{i=0}^{\infty} [\beta\delta(1-\lambda\chi)]^i \, \hat{r}_{t+i}. \tag{36}$$

It becomes intuitive that HANK can either resolve or amplify the FG puzzle by observing the second term on the RHS.

To see it more clearly, we're going to do some more recursion – but this time start with (33),

$$\hat{C}_t = \delta \mathbb{E}_t \hat{C}_{t+1} - \frac{1-\lambda}{\sigma(1-\lambda\chi)} \hat{r}_t.$$

Substitute $\hat{C}_{t+1}$, $\hat{C}_{t+2}$, and so on into the above expression to get

$$\begin{aligned}
\hat{C}_t &= \delta \mathbb{E}_t \left[\delta \mathbb{E}_{t+1} \hat{C}_{t+2} - \frac{1-\lambda}{\sigma(1-\lambda\chi)} \hat{r}_{t+1} \right] - \frac{1-\lambda}{\sigma(1-\lambda\chi)} \hat{r}_t \\
&= \delta^2 \mathbb{E}_t \hat{C}_{t+2} - \delta \frac{1-\lambda}{\sigma(1-\lambda\chi)} \mathbb{E}_t \hat{r}_{t+1} - \frac{1-\lambda}{\sigma(1-\lambda\chi)} \hat{r}_t \\
&= \delta^2 \mathbb{E}_t \left[\delta \mathbb{E}_{t+2} \hat{C}_{t+3} - \frac{1-\lambda}{\sigma(1-\lambda\chi)} \hat{r}_{t+2} \right] - \delta \frac{1-\lambda}{\sigma(1-\lambda\chi)} \mathbb{E}_t \hat{r}_{t+1} - \frac{1-\lambda}{\sigma(1-\lambda\chi)} \hat{r}_t \\
&\vdots \\
&= \delta^T \mathbb{E}_t \hat{C}_{t+T} - \mathbb{E}_t \sum_{k=1}^{T} \delta^{k-1} \frac{1-\lambda}{\sigma(1-\lambda\chi)} \hat{r}_{t+k-1},
\end{aligned}$$

or for any k from 0 to T, where k is the date of the FG announcement and T is the date of implementation of the interest rate change:

$$\hat{C}_{t+k} = \delta^{T-k} \mathbb{E}_{t+k} \hat{C}_{t+T} - \mathbb{E}_t \sum_{k=0}^{T} \delta^{T-k} \frac{1-\lambda}{\sigma(1-\lambda\chi)} \hat{r}_{t+k}.$$

Then for any k from 0 to T, the total effect of FG is

$$\Omega^{F(k)} \equiv \frac{d\hat{C}_{t+k}}{d(-r_{t+T})} = \delta^{T-k} \frac{1-\lambda}{\sigma(1-\lambda\chi)}.$$

The direct FG effect is obtained by differentiating (36):

$$\Omega_D^F \equiv \left. \frac{d\hat{C}_{t+k}}{d(-\hat{r}_{t+T})} \right|_{\hat{Y}_{t+k}=\bar{Y}} = \frac{(1-\lambda)\beta}{\sigma} \left[\delta\beta \left(1-\lambda\chi\right) \right]^T.$$

The indirect FG effect can be retrieved from the first term of (36):

$$\begin{aligned}
\Omega_D^F \equiv \left.\frac{d\hat{C}_{t+k}}{d(-\hat{r}_{t+T})}\right|_{\hat{r}_{t+k}=\bar{r}} &= [1-\beta(1-\lambda\chi)]\,\mathbb{E}_{t+k}\sum_{i=0}^{T}[\beta\delta(1-\lambda\chi)]^i\,\frac{d\hat{C}_{t+i}}{d(-\hat{r}_{t+T})} \\
&= \frac{1-\lambda}{\sigma(1-\lambda\chi)}[1-\beta(1-\lambda\chi)]\,\mathbb{E}_{t+k}\sum_{i=0}^{T}[\beta\delta(1-\lambda\chi)]^i\,\delta^{T-i} \\
&= \frac{1-\lambda}{\sigma(1-\lambda\chi)}\delta^T\left\{1-[\beta(1-\lambda\chi)]^{1+T}\right\}.
\end{aligned}$$

This gives:

Proposition (Bilbiie (2020) Proposition 4)**.** *The multiplier of FG (an interest rate cut in T periods) and the MPC in the analytical HANK model are*

$$\Omega^F = \frac{1-\lambda}{\sigma(1-\lambda\chi)}\delta^T, \quad \omega^F = 1-[\beta(1-\lambda\chi)]^{1+T}.$$

The multiplier decreases with the horizon $\partial\Omega^F/\partial T < 0$, thus resolving the FG puzzle, if and only if there is discounting $\delta < 1$. While in the compounding case, the multiplier increases with the horizon $\partial\Omega^F/\partial T > 0$ and the FG puzzle is aggravated.

In RANK ($s = 1$ and $\lambda = 0$), $\Omega^F = 1$ and is invariant to time – the classic case of the FG puzzle (Del Negro, Giannoni, and Patterson, 2023); an interest rate cut announcement has the same effect regardless of whether it takes place soon or much later – vastly at odds with the empirical evidence. In the TANK limit $s = h = 1$ with $\delta = 1$, contemporaneously, FG is more ($\chi > 1$) or less ($\chi < 1$) powerful than in RANK, but still time invariant. In other words, the potency of FG does not depend on T and so the puzzle remains in TANK.

But HANK resolves the FG puzzle and in a pretty intuitive way (with discounting). Since agents are unsure about whether or not they can benefit from the rate cut (due to being hand-to-mouth), the further away the rate cut is there, the less it leads to an increase in consumption. In other words, with discounting, the power of FG decreases over the time horizon as shown by McKay, Nakamura, and Steinsson (2016) first showed in a special case nested here for $\chi = 0$ and IID idiosyncratic uncertainty $1 - s = \lambda$. Bilbiie's proposition here is that this applies generally as long as there is some idiosyncratic uncertainty ($1-s > 0$) and fiscal redistribution or whatever else makes $\chi < 1$. These lead to precautionary savings and thus an under-reaction with respect to RANK and TANK in response to good future income news such as FG.

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