PhD Macroeconomics:
The New Keynesian Model

David Murakami

University of Milan and University of Pavia

24 April 2024

Introduction

- In the RBC model, everything is in real terms. There is no role for money (i.e., money is neutral).
- But there is a lot of empirical evidence to suggest that monetary policy influences short-run economic fluctuations.
- Need to merge classic Keynesian ideas of price stickiness with the RBC DSGE framework.
- New Keynesian theory attempted to provide "microfoundations" and to formalise Keynesian concepts.
- The synthesis of New Keynesian economics and the RBC theory resulted in the "new neoclassical synthesis" (Goodfriend and King, 1997) or the "neo-Wicksellian" approach (Woodford, 2003).
- In these slides I will do my best to stick to the notation of Galí (2015).

A Baseline New Keynesian Model

Households I

- The representative household derives utility from consumption C_t of final goods and disutility from labour N_t:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t; Z_t\right),$$

where $\beta \in (0, 1)$ is the household discount factor and the utility function is "well behaved" and satisfies the usual Inada condition.

- The period utility function is given by:

$$u\left(C_t, N_t; Z_t\right) = \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \varphi_0 \frac{N_t^{1+\varphi}}{1+\varphi}\right] Z_t,$$

where σ is the Arrow-Pratt coefficient of relative risk aversion and φ is the inverse-Frisch elasticity of labour supply.

Households II

- Z_t follows a stationary AR(1) process:

$$z_t \equiv \ln Z_t = \rho_z z_{t-1} + \varepsilon_t^z.$$

- The HH nominal flow budget constraint is:

$$P_t C_t + B_t \leq W_t N_t + R_{t-1} B_{t-1} + D_t,$$

where money is the numeraire, P_t: price of goods in terms of money, B_t: stock of nominal risk-free one-period bonds, and $R_t = 1 + i_t$: gross nominal interest rate.

- Households earn nominal profits D_t remitted to them by firms.

Households III

- Using our timing trick from when we solved the RBC model, the Lagrangian for the household is:

$$\mathcal{L} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \varphi_0 \frac{N_{t+s}^{1+\varphi}}{1+\varphi} \right] Z_{t+s}$$
$$+ \lambda_t \left(W_t N_t + D_t + R_{t-1} B_{t-1} - P_t C_t - B_t \right)$$
$$+ \beta \mathbb{E}_t \lambda_{t+1} \left[W_{t+1} N_{t+1} + D_{t+1} + R_t B_t - P_{t+1} C_{t+1} - B_{t+1} \right].$$

Households IV

- The FOCs – which look very familiar – are:

$$\frac{\partial \mathcal{L}}{\partial C_t} = C_t^{-\sigma} Z_t - \lambda_t P_t = 0, \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = -\varphi_0 N_t^{\varphi} Z_t + \lambda_t W_t = 0, \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial B_t} = -\lambda_t + \beta \mathbb{E}_t \lambda_{t+1} R_t = 0. \quad (3)$$

- From (3) we know that $\lambda_t = \beta \mathbb{E}_t \lambda_{t+1} R_t$, so we can write the FOCs as:

$$C_t^{-\sigma} Z_t = P_t \beta \mathbb{E}_t \lambda_{t+1} R_t, \quad (4)$$

$$\varphi_0 N_t^{\varphi} Z_t = \lambda_t W_t. \quad (5)$$

Households V

- Then, from (4), like we do in the RBC models, we have:

$$\frac{C_t^{-\sigma}}{P_t} Z_t = \beta \mathbb{E}_t \lambda_{t+1} R_t = \lambda_t,$$

so we can roll one period ahead to get:

$$\frac{C_{t+1}^{-\sigma}}{P_{t+1}} Z_{t+1} = \lambda_{t+1},$$

and combining (4) and (5) we can get rid of λ_t from our FOCs:

$$\varphi_0 N_t^{\varphi} = \frac{C_t^{-\sigma}}{P_t} W_t, \tag{6}$$

$$C_t^{-\sigma} = \frac{P_t}{Z_t} \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{P_{t+1}} Z_{t+1} R_t = \beta \mathbb{E}_t C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} \frac{Z_{t+1}}{Z_t} R_t. \tag{7}$$

Firms and production I

- Split production into two sectors.
- There is a representative perfectly competitive final good firm, and monopolistically competitive intermediate goods firms.
- The final output good is a CES aggregate, utilising the Dixit-Stiglitz aggregator, of a continuum of intermediate goods:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 0,$$

so final good firms maximise their profits by selecting how much of each intermediate good to purchase, and so their problem is:

$$\max_{Y_t(j)} \left\{ P_t Y_t - \int_0^1 P_t Y_t(j) dj \right\}.$$

Firms and production II

- The FOC for a typical intermediate good j is:

$$Y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\epsilon} Y_t. \tag{8}$$

- From Blanchard and Kiyotaki (1987), we can derive a price index for the aggregate economy:

$$P_t Y_t \equiv \int_0^1 P_t(j) Y_t(j) dj.$$

- Then, plugging in the demand for good j from (8) we have:

$$\Longrightarrow P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}. \tag{9}$$

Intermediate firms I

- A typical intermediate firm produces output according a constant returns to scale technology in labour, with a common productivity shock, A_t:

$$Y_t(j) = A_t N_t(j)^{1-\alpha}. \tag{10}$$

- Intermediate firms pay a common wage.
- They are not freely able to adjust price so as to maximise profit each period, but will always act to minimise cost.

Intermediate firms II

- The cost minimisation problem is to minimise total cost subject to the constraint producing enough to meet demand:

$$\min_{N_t(j)} \; W_t N_t(j),$$

subject to

$$A_t N_t(j)^{1-\alpha} \geq Y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\epsilon} Y_t.$$

Intermediate firms III

- The Lagrangian for an intermediate firm j's problem is:

$$\mathcal{L} = W_t N_t(j) - \Psi_t(j) \left(A_t N_t(j)^{1-\alpha} - \left[\frac{P_t(j)}{P_t} \right]^{-\epsilon} Y_t \right),$$

where $\Psi_t(j)$ is the Lagrangian multiplier for firm j. The FOC is:

$$\frac{\partial \mathcal{L}}{\partial N_t(j)} = W_t - (1-\alpha)\Psi_t(j) A_t N_t(j)^{-\alpha} = 0,$$

which then implies:

$$\Psi_t(j) = \frac{W_t}{(1-\alpha)A_t} N_t(j)^{\alpha} = \frac{W_t}{(1-\alpha)A_t} \left[\frac{Y_t(j)}{A_t} \right]^{\frac{\alpha}{1-\alpha}}. \tag{11}$$

Intermediate firms IV

- Notice that when $\alpha = 0$ neither W_t nor A_t are firm j specific, so in fact we can write $\Psi_t(j)$ as simply Ψ_t.
- Now, what is the economic interpretation of $\Psi_t(j)$? It is an intermediate firm's nominal marginal cost – how much costs change if you are forced to produce an extra unit of output.
- For the case where $\alpha \neq 0$, define the economy-wide average marginal cost as

$$\Psi_t = \frac{W}{(1-\alpha)A_t}\left(\frac{Y_t}{A_t}\right)^{\frac{\alpha}{1-\alpha}} . \tag{12}$$

- Then we can write:

$$\Psi_t(j) = \Psi_t\left[\frac{Y_t}{Y_t(j)}\right]^{\frac{-\alpha}{1-\alpha}} = \Psi_t\left[\frac{P_t(j)}{P_t}\right]^{\frac{\epsilon\alpha}{\alpha-1}} , \tag{13}$$

which shows that firms with high prices have low marginal costs.

Intermediate firms V

- Now, formulate the intermediate firm's real flow profit as:

$$\frac{D_t(j)}{P_t} = \frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t}{P_t} N_t(j),$$

and substitute in the nominal wage from (11):

$$\begin{aligned} \frac{D_t(j)}{P_t} &= \frac{P_t(j)}{P_t} Y_t(j) - (1-\alpha) \frac{\Psi_t(j) A_t}{P_t} N_t(j)^{1-\alpha} \\ &= \frac{P_t(j)}{P_t} Y_t(j) - MC_t Y_t(j), \end{aligned} \tag{14}$$

where $MC_t(j) = \frac{\Psi_t(j)}{P_t}$ is the real marginal cost for an intermediate firm.

- Now, buckle up because this is where the fun begins...

Monopolistic competition with Calvo pricing I

- Intermediate firms are not able to freely adjust prices each period.
- Each period they are unable to adjust prices wp θ. Conversely, they can adjust prices in any given period wp $1-\theta$.
  - They get a visit from the "Calvo fairy" wp $1-\theta$.
- So, the probability a firm is stuck with today's price s periods ahead is θ^s.
- Firms will discount s periods into the future by:

$$\Lambda_{t,t+s}\theta^s,$$

where

$$\Lambda_{t,t+s} = \beta^s \frac{u_{C,t+s}}{u_{C,t}},$$

is the household stochastic discount factor (SDF).

Monopolistic competition with Calvo pricing II

- The dynamic problem of an updating firm can be written as

$$\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\theta^s \left(\frac{P_t(j)}{P_{t+s}} \underbrace{\left[\frac{P_t(j)}{P_{t+s}}\right]^{-\epsilon} Y_{t+s}}_{Y_{t+s}(j)} - MC_{t+s}(j) \underbrace{\left[\frac{P_t(j)}{P_{t+s}}\right]^{-\epsilon} Y_{t+s}}_{Y_{t+s}(j)} \right), \tag{15}$$

  where we assume that output will equal demand.
- Expanding the terms, we get

$$\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \left(\Lambda_{t,t+s}\theta^s P_t(j)^{1-\epsilon} P_{t+s}^{\epsilon-1} Y_{t+s} - \Lambda_{t,t+s}\theta^s MC_{t+s}(j)(j) P_t(j)^{-\epsilon} P_{t+s}^{\epsilon} Y_{t+s}\right),$$

Monopolistic competition with Calvo pricing III

and so the FOC is:

$$0 = (1-\epsilon)P_t(j)^{-\epsilon}\mathbb{E}_t\sum_{s=0}^{\infty}\Lambda_{t,t+s}\theta^s P_{t+s}^{\epsilon-1}Y_{t+s} + \epsilon P_t(j)^{-\epsilon-1}\mathbb{E}_t\sum_{s=0}^{\infty}\Lambda_{t,t+s}\theta^s MC_{t+s}(j)P_{t+s}^{\epsilon}Y_{t+s}.$$

- Move the first summation to the LHS and do some algebra to write:

$$P_t(j) = \frac{\epsilon}{\epsilon-1}\frac{\mathbb{E}_t\sum_{s=0}^{\infty}\beta^s u_{C,t+s}\theta^s MC_{t+s}(j)P_{t+s}^{\epsilon}Y_{t+s}}{\mathbb{E}_t\sum_{s=0}^{\infty}\beta^s u_{C,t+s}\theta^s P_{t+s}^{\epsilon-1}Y_{t+s}}.$$

Monopolistic competition with Calvo pricing IV

- Use (13) (adjust appropriately to get MC_t) to write the above expression as:

$$P_t(j) = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_{C,t+s} \theta^s MC_t \left[\frac{P_t(j)}{P_{t+s}}\right]^{\frac{\epsilon\alpha}{\alpha-1}} P_{t+s}^{\epsilon} Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_{C,t+s} \theta^s P_{t+s}^{\epsilon-1} Y_{t+s}}$$

$$P_t(j)^{1+b} = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_{C,t+s} \theta^s MC_t P_{t+s}^{\epsilon + \frac{\epsilon\alpha}{1-\alpha}} Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_{C,t+s} \theta^s P_{t+s}^{\epsilon-1} Y_{t+s}},$$

  where $b = \frac{\epsilon\alpha}{1-\alpha}$.
- Note that none of the variables on the RHS depend on j.

Monopolistic competition with Calvo pricing V

- This means that any firm able to update their price will update to a common optimal price, say, $P_t(j) = P_t^*$. Write this compactly as:

$$(P_t^*)^{1+b} = \mathcal{M}\frac{X_{1,t}}{X_{2,t}}, \tag{16}$$

where $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$ is the optimal markup charged by monopolistically competitive firms, and our auxiliary variables $X_{1,t}$ and $X_{2,t}$ are:

$$X_{1,t} = u_{C,t}MC_tP_t^{\epsilon+b}Y_t + \theta\beta\mathbb{E}_tX_{1,t+1}, \tag{17}$$

$$X_{2,t} = u_{C,t}P_t^{\epsilon-1}Y_t + \theta\beta\mathbb{E}_tX_{2,t+1}. \tag{18}$$

- Notice that the second terms of our auxiliary variables are equal to 0 when $\theta = 0$, i.e., if all firms are able to change their prices freely, then prices are flexible which means that $MC_tP_t = \Psi_t$ and $\Psi_t = \mathcal{M}^{-1}$. This is important to note down.

Equilibrium and aggregation I

- To close the model, we need to specify an exogenous process for our technology shocks A_t and some kind of monetary policy rule to determine M_t.
- As before, let the aggregate productivity term follow an AR(1) process such as:

$$a_t \equiv \ln A_t = \rho_a a_{t-1} + \varepsilon_t^a. \tag{19}$$

- Then, let's suppose that demand for real money balances is given by the following ad-hoc equation:

$$\frac{M_t}{P_t} = \frac{Y_t}{R_t^\eta} \tag{20}$$

$$\Leftrightarrow m_t - p_t = y_t - \eta i_t.$$

Equilibrium and aggregation II

- Suppose that the nominal money supply also follows an AR(1) process in the growth rate:

$$\Delta m_t \equiv \Delta \ln M_t = (1 - \rho_m)\pi + \rho_m \Delta m_{t-1} + \varepsilon_t^m, \tag{21}$$

where $\Delta m_t \equiv m_t - m_{t-1}$.

- Mean growth rate of money is equal to the steady state net inflation rate π, as we want real money balances to be stationary.
- For both the law of motion of technology and nominal money, I assume that they contain white noise shock terms such that $\varepsilon_t^a \stackrel{\text{IID}}{\sim} \mathcal{N}(0, \sigma_a^2)$ and $\varepsilon_t^m \stackrel{\text{IID}}{\sim} \mathcal{N}(0, \sigma_m^2)$.

Equilibrium and aggregation III

- In equilibrium, bond-holding is always zero in all periods: $B_t = 0$. Using this, the household budget constrain can be written in real terms:

$$P_t C_t + B_t \leq W_t N_t + R_{t-1} B_{t-1} + D_t$$
$$\Leftrightarrow C_t = \frac{W_t N_t}{P_t} + \frac{D_t}{P_t}.$$

- Real dividends received by the household are just the sum of real profits from intermediate goods firms (since the final good firm is competitive and earns no economic profit):

$$\frac{D_t}{P_t} = \int_0^1 \left[\frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t}{P_t} N_t(j) \right] dj$$
$$= \int_0^1 \frac{P_t(j)}{P_t} Y_t(j) dj - \frac{W_t}{P_t} N_t$$

Equilibrium and aggregation IV

- So, the household budget constraint becomes:

$$C_t = \frac{W_t N_t}{P_t} + \int_0^1 \frac{P_t(j)}{P_t} Y_t(j) dj - \frac{W_t N_t}{P_t}$$

$$\implies C_t = \int_0^1 \frac{P_t(j)}{P_t} Y_t(j) dj,$$

and since:

$$Y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\epsilon} Y_t,$$

Equilibrium and aggregation V

we have:

$$C_t = \int_0^1 \frac{P_t(j)}{P_t} \left[\frac{P_t(j)}{P_t} \right]^{-\epsilon} Y_t dj$$
$$= \int_0^1 P_t(j)^{1-\epsilon} P_t^{\epsilon-1} Y_t dj$$
$$= P_t^{\epsilon-1} Y_t \int_0^1 P_t(j)^{1-\epsilon} dj,$$

but $\int_0^1 P_t(j)^{1-\epsilon} dj = P_t^{1-\epsilon}$ from (9), so the P_t terms drop out and we have the market clearing condition:

$$C_t = Y_t \tag{22}$$

Equilibrium and aggregation VI

- Now, we need to solve for Y_t. But we first need to get aggregate labour demand by firms, N_t: (8) and (10) give:

$$N_t(j) = \left[\frac{Y_t(j)}{A_t}\right]^{\frac{1}{1-\alpha}} .$$

Aggregate this across firms to then get

$$N_t \equiv \int_0^1 N_t(j)dj = \int_0^1 \left[\frac{Y_t(j)}{A_t}\right]^{\frac{1}{1-\alpha}} dj,$$

then substitute in for $Y_t(j)$ using the demand for intermediate goods:

$$N_t = \int_0^1 \left\{ \frac{\left[\frac{P_t(j)}{P_t}\right]^{-\epsilon} Y_t}{A_t} \right\}^{\frac{1}{1-\alpha}} dj. \tag{23}$$

Equilibrium and aggregation VII

- Now, do a bit of algebra and solve for Y_t:

$$Y_t = \frac{A_t N_t^{1-\alpha}}{\underbrace{\left\{ \int_0^1 \left[\frac{P_t(j)}{P_t} \right]^{-\frac{\epsilon}{1-\alpha}} dj \right\}^{1-\alpha}}_{V_t^P}} = \frac{A_t N_t^{1-\alpha}}{V_t^P}. \tag{24}$$

- The new variable we have defined, V_t^P, is a measure of "price dispersion".
- If there were no pricing frictions, all firms would charge the same price, and $V_t^P = 1$.
- Since $V_t^P \geq 1$, price dispersion leads to lower output. This is the gist for why price stability is a good thing.

Full set of equilibrium conditions I

$$
\begin{aligned}
C_t^{-\sigma} &= \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma} R_t P_t Z_{t+1}}{P_{t+1} Z_t}, \\
\varphi_0 N_t^{\varphi} &= C_t^{-\sigma} \frac{W_t}{P_t}, \\
MC_t &= \frac{W_t / P_t}{(1-\alpha) A_t (Y_t / A_t)^{\frac{-\alpha}{1-\alpha}}}, \\
C_t &= Y_t, \\
Y_t &= \frac{A_t N_t^{1-\alpha}}{V_t^P}, \\
V_t^P &= \left\{ \int_0^1 \left[\frac{P_t(j)}{P_t} \right]^{-\frac{\epsilon}{1-\alpha}} dj \right\}^{1-\alpha},
\end{aligned}
$$

Full set of equilibrium conditions II

$$
\begin{aligned}
P_t^{1-\epsilon} &= \int_0^1 P_t(j)^{1-\epsilon} dj, \\
(P_t^*)^{1+\frac{\epsilon\alpha}{1-\alpha}} &= \mathcal{M}\frac{X_{1,t}}{X_{2,t}}, \\
X_{1,t} &= C_t^{-\sigma} Z_t MC_t P_t^{\epsilon+\frac{\epsilon\alpha}{1-\alpha}} Y_t + \theta\beta\mathbb{E}_t X_{1,t+1}, \\
X_{2,t} &= C_t^{-\sigma} Z_t P_t^{\epsilon-1} Y_t + \theta\beta\mathbb{E}_t X_{2,t+1}, \\
\frac{M_t}{P_t} &= \frac{Y_t}{R_t^{\eta}}, \\
\ln Z_t &= \rho_z \ln Z_{t-1} + \varepsilon_t^z, \\
\ln A_t &= \rho_a \ln A_{t-1} + \varepsilon_t^a, \\
\Delta m_t &= (1-\rho_m)\pi + \rho_m \Delta m_{t-1} + \varepsilon_t^m,
\end{aligned}
$$

Full set of equilibrium conditions III

- This is 14 equations in 14 aggregate variables. But there are three issues with the way we have written up this system of equations:
  - We have heterogeneity (j shows up);
  - The price level shows up and it isn't stationary;
  - Nominal money growth shows up and it isn't stationary.
- So we will rewrite these conditions using Calvo pricing to get rid of the j terms, and use inflation instead of price levels.
- We also need to ensure that trending variables are detrended.

Rewriting the equilibrium conditions I

- Begin by rewriting gross inflation as $\Pi_t = 1 + \pi_t = \frac{P_t}{P_{t-1}}$. The consumption Euler equation can be re-written as:

$$C_t^{-\sigma} = \beta \mathbb{E}_t C_{t+1}^{-\sigma} R_t \Pi_{t+1} \frac{Z_{t+1}}{Z_t}.$$

- The demand for money equation (20) is already written in terms of real money balances, M_t/P_t, which is stationary so it's fine.
- Now we need to get rid of the j terms in the price level and price dispersion expansions. The expression for the price level is:

$$P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} dj.$$

Rewriting the equilibrium conditions II

- Now, recall that a fraction $1-\theta$ of these firms will update their price to the same optimal price, P_t^*.
- The other fraction θ will charge the price they charged in the previous period.
- Since it doesn't matter how we "order" these firms along the unit interval, this means we can break up the integral on the RHS above as:

$$P_t^{1-\epsilon} = \int_0^{1-\theta} \left(P_t^*\right)^{1-\epsilon} dj + \int_{1-\theta}^{1} P_{t-1}(j)^{1-\epsilon} dj$$

$$\Leftrightarrow P_t^{1-\epsilon} = (1-\theta)\left(P_t^*\right)^{1-\epsilon} + \int_{1-\theta}^{1} P_{t-1}(j)^{1-\epsilon} dj.$$

Rewriting the equilibrium conditions III

- Now, watch the Calvo magic:

$$\int_{1-\theta}^{1} P_t(j)^{1-\epsilon} dj = \theta \int_0^1 P_{t-1}(j)^{1-\epsilon} dj = \theta P_{t-1}^{1-\epsilon},$$

  and therefore we have

$$P_t^{1-\epsilon} = (1-\theta)\left(P_t^*\right)^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon}.$$

- Tada! We've gotten rid of the heterogeneity. The Calvo assumption allows us to integrate out the heterogeneity.

Rewriting the equilibrium conditions IV

- Now, we want to write things in terms of inflation, so divide both sides by $P_{t-1}^{1-\epsilon}$, and define $\Pi_t^* = 1 + \pi_t^* = \frac{P_t^*}{P_{t-1}}$ as "optimal price inflation":

$$\begin{aligned} \frac{P_t^{1-\epsilon}}{P_{t-1}^{1-\epsilon}} &= (1-\theta)\frac{(P_t^*)^{1-\epsilon}}{P_{t-1}^{1-\epsilon}} + \theta\frac{P_{t-1}^{1-\epsilon}}{P_{t-1}^{1-\epsilon}} \\ \Leftrightarrow \Pi_t^{1-\epsilon} &= (1-\theta)(\Pi_t^*)^{1-\epsilon} + \theta. \end{aligned} \tag{25}$$

Rewriting the equilibrium conditions V

- Now, look at the price dispersion term. Notice we can use the same Calvo trick we used above here:

$$
\begin{aligned}
(V_t^P)^{\frac{1}{1-\alpha}} &= \int_0^1 \left[\frac{P_t(j)}{P_t}\right]^{-\frac{\epsilon}{1-\alpha}} dj = \int_0^{1-\theta} \left[\frac{P_t(j)}{P_t}\right]^{-\frac{\epsilon}{1-\alpha}} dj + \int_{1-\theta}^1 \left[\frac{P_{t-1}(j)}{P_t}\right]^{-\frac{\epsilon}{1-\alpha}} dj \\
&= \int_0^{1-\theta} \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} dj + \int_{1-\theta}^1 \left[\frac{P_{t-1}(j)}{P_t}\right]^{-\frac{\epsilon}{1-\alpha}} dj \\
&= \int_0^{1-\theta} \left(\frac{P_t^*}{P_{t-1}}\right)^{-\frac{\epsilon}{1-\alpha}} \left(\frac{P_{t-1}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} dj + \int_{1-\theta}^1 \left[\frac{P_{t-1}(j)}{P_{t-1}}\right]^{-\frac{\epsilon}{1-\alpha}} \left(\frac{P_{t-1}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} dj \\
&= (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{-\frac{\epsilon}{1-\alpha}} \left(\frac{P_t}{P_{t-1}}\right)^{\frac{\epsilon}{1-\alpha}} + \left(\frac{P_t}{P_{t-1}}\right)^{\frac{\epsilon}{1-\alpha}} \int_{1-\theta}^1 \left[\frac{P_{t-1}(j)}{P_{t-1}}\right]^{-\frac{\epsilon}{1-\alpha}} dj \\
&= (1-\theta)(\Pi_t^*)^{-\frac{\epsilon}{1-\alpha}} \Pi_t^{\frac{\epsilon}{1-\alpha}} + \Pi_t^{\frac{\epsilon}{1-\alpha}} \int_{1-\theta}^1 \left[\frac{P_{t-1}(j)}{P_{t-1}}\right]^{-\frac{\epsilon}{1-\alpha}} dj,
\end{aligned}
$$

Rewriting the equilibrium conditions VI

and use the Calvo trick, and the definition of price dispersion, on the last term of the RHS to get:

$$(V_t^P)^{\frac{1}{1-\alpha}} = (1-\theta)(\Pi_t^*)^{-\frac{\epsilon}{1-\alpha}} \Pi_t^{\frac{\epsilon}{1-\alpha}} + \theta \Pi_t^{\frac{\epsilon}{1-\alpha}} (V_{t-1}^P)^{\frac{1}{1-\alpha}}. \tag{26}$$

- Now, adjust the reset price expression. First, divide the following auxiliary variables by P_t raised to the relevant powers:

$$\frac{X_{1,t}}{P_t^{\epsilon+b}} = \frac{C_t^{-\sigma} Z_t MC_t P_t^{\epsilon+b} Y_t}{P_t^{\epsilon+b}} + \theta\beta \frac{\mathbb{E}_t X_{1,t+1}}{P_t^{\epsilon+b}},$$
$$\frac{X_{2,t}}{P_t^{\epsilon-1}} = \frac{C_t^{-\sigma} Z_t P_t^{\epsilon-1} Y_t}{P_t^{\epsilon-1}} + \theta\beta \frac{\mathbb{E}_t X_{2,t+1}}{P_t^{\epsilon-1}}$$

where $b = \epsilon\alpha/(1-\alpha)$.

Rewriting the equilibrium conditions VII

- Multiplying and dividing the second terms on the RHS by P_{t+1} to the right powers yields:

$$\frac{X_{1,t}}{P_t^{\epsilon+b}} = C_t^{-\sigma} Z_t MC_t Y_t + \theta\beta\mathbb{E}_t \frac{X_{1,t+1}}{P_{t+1}^{\epsilon+b}} \left(\frac{P_{t+1}}{P_t}\right)^{\epsilon+b}$$

$$= C_t^{-\sigma} Z_t MC_t Y_t + \theta\beta\mathbb{E}_t \Pi_{t+1}^{\epsilon+b} \frac{X_{1,t+1}}{P_{t+1}^{\epsilon+b}},$$

$$\frac{X_{2,t}}{P_t^{\epsilon-1}} = C_t^{-\sigma} Z_t Y_t + \theta\beta\mathbb{E}_t \frac{X_{2,t+1}}{P_{t+1}^{\epsilon-1}} \left(\frac{P_{t+1}}{P_t}\right)^{\epsilon-1}$$

$$= C_t^{-\sigma} Z_t Y_t + \theta\beta\mathbb{E}_t \Pi_{t+1}^{\epsilon-1} \frac{X_{2,t+1}}{P_{t+1}^{\epsilon-1}}.$$

Rewriting the equilibrium conditions VIII

- So the reset price expression can now be written as:

$$(P_t^*)^{1+b} = \mathcal{M}\frac{X_{1,t}/P_t^{\epsilon+b}}{X_{2,t}/P_t^{\epsilon-1}}P_t^{1+b},$$

and by dividing both sides by P_{t-1}^{1+b} we can write this in terms of inflation:

$$(\Pi_t^*)^{1+b} = \mathcal{M}\frac{X_{1,t}/P_t^{\epsilon+b}}{X_{2,t}/P_t^{\epsilon-1}}\Pi_t^{1+b}. \tag{27}$$

Rewriting the equilibrium conditions IX

- The process for real money balances can be converted into real terms quite easily:

$$\Delta m_t = m_t - m_{t-1},$$

and then do some add and subtraction:

$$\begin{aligned}\Delta m_t &= m_t - m_{t-1} + p_t - p_t + p_{t-1} - p_{t-1} \\ &= m_t - p_t - (m_{t-1} - p_{t-1}) + p_t - p_{t-1} \\ &= \Delta(m_t - p_t) + \pi_t.\end{aligned}$$

- If you really want to, you could make some new auxiliary variables to easily track the real variables from their nominal counterparts.
  - For example, you could define, say, $M_t^r = M_t / P_t$ or $\tilde{X}_{1,t} = X_{1,t}/P_t^{\epsilon+b}$.

Rewriting the equilibrium conditions X

- So we can write the process for money growth in terms of real balance growth as:

$$\Delta(m_t - p_t) + \pi_t = (1 - \rho_m)\pi + \rho_m \Delta(m_{t-1} - p_{t-1}) + \rho_m \pi_{t-1} + \varepsilon_t^m . \tag{28}$$

Rewriting the equilibrium conditions XI

- The full set of rewritten equilibrium conditions is:

$$
\begin{aligned}
C_t^{-\sigma} &= \beta \mathbb{E}_t C_{t+1}^{-\sigma} \Pi_{t+1} R_t \frac{Z_{t+1}}{Z_t}, \\
\varphi_0 N_t^{\varphi} &= C_t^{-\sigma} \frac{W_t}{P_t}, \\
MC_t &= \frac{W_t/P_t}{(1-\alpha) A_t (Y_t/A_t)^{\frac{-\alpha}{1-\alpha}}}, \\
C_t &= Y_t, \\
Y_t &= \frac{A_t N_t^{1-\alpha}}{V_t^P}, \\
(V_t^P)^{\frac{1}{1-\alpha}} &= (1-\theta)(\Pi_t^*)^{-\frac{\epsilon}{1-\alpha}} \Pi_t^{\frac{\epsilon}{1-\alpha}} + \theta \Pi_t^{\frac{\epsilon}{1-\alpha}} (V_{t-1}^P)^{\frac{1}{1-\alpha}},
\end{aligned}
$$

Rewriting the equilibrium conditions XII

$$
\begin{aligned}
\Pi_t^{1-\epsilon} &= (1-\theta)(\Pi_t^*)^{1-\epsilon} + \theta, \\
(\Pi_t^*)^{1+b} &= \mathcal{M}\frac{X_{1,t}/P_t^{\epsilon+b}}{X_{2,t}/P_t^{\epsilon-1}}\Pi_t^{1+b}, \\
\frac{X_{1,t}}{P_t^{\epsilon+b}} &= C_t^{-\sigma}MC_tY_t + \theta\beta\mathbb{E}_t\Pi_{t+1}^{\epsilon+b}\frac{X_{1,t+1}}{P_{t+1}^{\epsilon+b}}, \\
\frac{X_{2,t}}{P_t^{\epsilon-1}} &= C_t^{-\sigma}Y_t + \theta\beta\mathbb{E}_t\Pi_{t+1}^{\epsilon-1}\frac{X_{2,t+1}}{P_{t+1}^{\epsilon-1}}, \\
\frac{M_t}{P_t} &= \frac{Y_t}{R_t^{\eta}}, \\
z_t &= \rho_z z_{t-1} + \varepsilon_t^z, \\
a_t &= \rho_a a_{t-1} + \varepsilon_t^a, \\
\Delta(m_t - p_t) + \pi_t &= (1-\rho_m)\pi + \rho_m\Delta(m_{t-1} - p_{t-1}) + \rho_m\pi_{t-1} + \varepsilon_t^m.
\end{aligned}
$$

The steady state I

- We now solve for the non-stochastic steady state of the model.
- We have $A = 1$, and since output and consumption are always equal, it must be that $Y = C$.
- Steady state inflation is equal to the exogenous target, π, which we will assume to be 0.
  - In other words, in what follows, we will assume that there is no trend inflation in the steady state. The case of trend inflation see Ascari (2004) and Ascari and Ropele (2009).

The steady state II

- Next, from the consumption Euler equation, we have:

$$\begin{aligned} C^{-\sigma} &= \beta C^{-\sigma} \Pi R \frac{Z}{Z} \\ \implies R &= \frac{1+\pi}{\beta} \\ \Leftrightarrow 1+i &= \frac{1+\pi}{\beta} \\ \implies i &= \rho + \pi, \end{aligned} \tag{29}$$

  where

$$\beta = \frac{1}{1+\rho}.$$

- (29) is the familiar Fisher equation, and ρ in the expression for β is the discount rate (whereas β is the discount factor), and is also referred to as the net real interest rate.

The steady state III

- From the price evolution equation, we can derive the steady state expression for reset price inflation:

$$\begin{aligned} \Pi^{1-\epsilon} &= (1-\theta)(\Pi^*)^{1-\epsilon} + \theta \\ \frac{\Pi^{1-\epsilon} - \theta}{1-\theta} &= (\Pi^*)^{1-\epsilon} \\ \implies \Pi^* &= \left(\frac{\Pi^{1-\epsilon} - \theta}{1-\theta} \right)^{\frac{1}{1-\epsilon}}. \end{aligned} \tag{30}$$

- If $\Pi = 1$, then $\Pi^* = \Pi$, since the RHS of the above expression is equal to 1. If $\Pi > 1 \implies \Pi^* > \Pi$, and if $\Pi < 1 \implies \Pi^* < \Pi$.

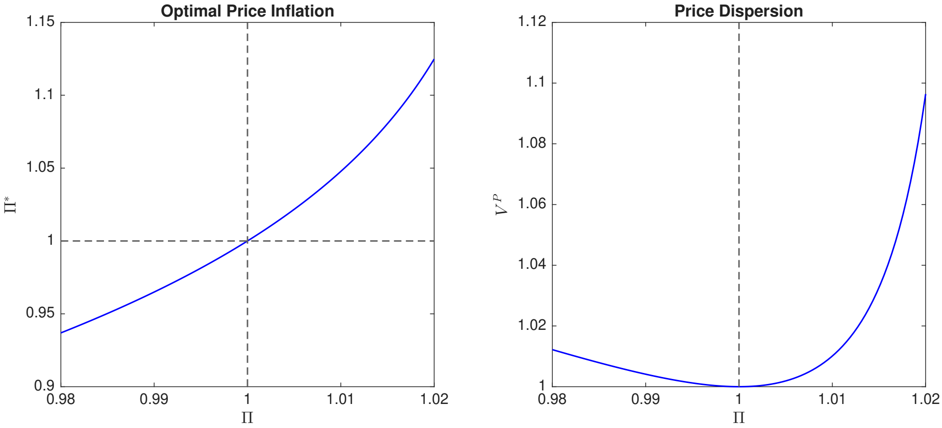
The steady state IV

- With this in hand, we can solve for steady state price dispersion:

$$
\begin{aligned}
(V^P)^{\frac{1}{1-\alpha}} &= (1-\theta)(\Pi^*)^{-\frac{\epsilon}{1-\alpha}}\Pi^{\frac{\epsilon}{1-\alpha}} + \theta\Pi^{\frac{\epsilon}{1-\alpha}}(V^P)^{\frac{1}{1-\alpha}} \\
\left(1-\theta\Pi^{\frac{\epsilon}{1-\alpha}}\right)(V^P)^{\frac{1}{1-\alpha}} &= \frac{(1-\theta)(\Pi)^{\frac{\epsilon}{1-\alpha}}}{(\Pi^*)^{\frac{\epsilon}{1-\alpha}}}. \qquad (31)
\end{aligned}
$$

- If $\Pi = 1$, then $V^P = 1$. If $\Pi \neq 1$, then $V^P > 1$.

The steady state V



NOTE: $\epsilon = 10$, $\theta = 0.75$, $\alpha = 0$.

The steady state VI

- Now, we can solve for the steady state ratio of $(X_1/P^{\epsilon+b})/(X_2/P^{\epsilon-1})$:

$$\frac{X_1/P^{\epsilon+b}}{X_2/P^{\epsilon-1}} = \mathcal{M}^{-1}\left(\frac{\Pi^*}{\Pi}\right)^{1+b}.$$

- Then take the equation for the auxiliary variable $X_{1,t}/P_t^{\epsilon+b}$, and do some rearranging:

$$\begin{aligned}\frac{X_1}{P^{\epsilon+b}}(1-\theta\beta\Pi^{\epsilon+b}) &= MC\frac{Y}{C}\\ \frac{X_1}{P^{\epsilon+b}} &= MC\frac{Y}{C}(1-\theta\beta\Pi^{\epsilon+b})^{-1},\end{aligned}$$

and do the same for $X_{2,t}/P_t^{\epsilon-1}$:

$$\frac{X_2}{P^{\epsilon-1}} = \frac{Y}{C}(1-\theta\beta\Pi^{\epsilon-1})^{-1}.$$

The steady state VII

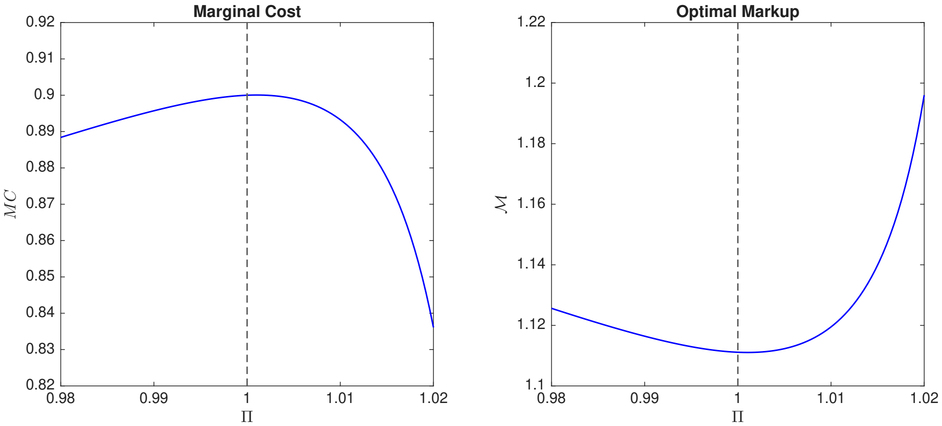
- So then we can divide the two:

$$\frac{X_1/P^{\epsilon+b}}{X_2/P^{\epsilon-1}} = MC\frac{1-\theta\beta\Pi^{\epsilon-1}}{1-\theta\beta\Pi^{\epsilon+b}}$$

$$MC = \mathcal{M}^{-1}\left(\frac{\Pi^*}{\Pi}\right)^{1+b}\frac{(1-\theta\beta\Pi^{\epsilon+b})}{(1-\theta\beta\Pi^{\epsilon-1})}.$$

- In words, the steady state real marginal cost is inverse to the price markup. If $\Pi = 1$, then $MC = \mathcal{M}^{-1} = \frac{\epsilon-1}{\epsilon}$. In other words, if steady state net inflation is zero, then the steady state markup will be what it would be if prices were fully flexible (also corresponding to $\theta = 0$).
- If $\Pi \neq 1$, then $MC < \frac{\epsilon-1}{\epsilon}$, which means that the steady state markup will be higher than it would be if net inflation were zero.

The steady state VIII



NOTE: $\epsilon = 10$, $\theta = 0.75$, $\alpha = 0$.

The steady state IX

- With the steady steady state marginal cost in hand, we now look at the labour supply condition. We have that

$$MC = \frac{W}{P} Y^{\frac{\alpha}{1-\alpha}},$$

since $A = 1$.

- The lower is the marginal cost, the bigger is the wedge between the wage and the marginal product of labour (i.e., the more distorted the economy is).
- Then we have:

$$\varphi_0 N^{\varphi} = \frac{W}{C^{\sigma} P}$$

$$\Leftrightarrow \varphi_0 N^{\varphi} = \frac{W}{Y^{\sigma} P},$$

The steady state X

and since $Y = \frac{AN^{1-\alpha}}{V^P}$, we have:

$$\begin{aligned}
\varphi_0 N^{\varphi} &= \frac{W}{P}\left(\frac{AN^{1-\alpha}}{V^P}\right)^{-\sigma} \\
&= \frac{W}{P}\left(V^P\right)^{\sigma} N^{-\sigma(1-\alpha)} \\
&= \frac{MC}{Y^{\frac{\alpha}{1-\alpha}}}\left(V^P\right)^{\sigma} N^{-\sigma(1-\alpha)} \\
&= MC(V^P)^{\sigma} N^{-\sigma(1-\alpha)}\left[\frac{N^{1-\alpha}}{V^P}\right]^{-\frac{\alpha}{1-\alpha}} \\
\varphi_0 N^{\sigma(1-\alpha)+\alpha+\varphi} &= MC(V^P)^{\frac{\sigma(1-\alpha)+\alpha}{1-\alpha}} \\
\therefore N &= \left[\frac{MC}{\varphi_0}(V^P)^{\frac{\sigma(1-\alpha)+\alpha}{1-\alpha}}\right]^{\frac{1}{\sigma(1-\alpha)+\alpha+\varphi}}.
\end{aligned}$$

The steady state XI

- Finally, since we have Y, steady state M/P is easy:

$$\frac{M}{P} = \frac{Y}{(1/\beta)^{\eta}}$$

The flexible price equilibrium I

- We now consider the hypothetical equilibrium case where all prices are flexible (i.e., when $\theta = 0$).
- Sometimes referred to as the "natural allocation".
- But even with flex prices, output is lower than an "RBC economy" since there is monopolistic competition.
- Let superscript n denote the hypothetical flex price allocation of a variable.
- When $\theta = 0$, firms will always price their goods optimally to P_t^*. Then, from the price dispersion equation we have:

$$V_t^{P,n} = \left(\frac{\Pi^*}{\Pi}\right)^{-\epsilon} = 1,$$

The flexible price equilibrium II

and combining this result with the (16) we have

$$\frac{P_t^*}{P_t} = \mathcal{M} MC_t(j).$$

- But when prices are flexible, the equilibrium is symmetric and we have: $P_t^* = P_t$, $MC_t(j) = \mathcal{M}^{-1}$, $Y_t(j) = Y_t$, and $N_t(j) = N_t$, $\forall j$.
- In words, if prices are flexible, all firms charge the same price, and price dispersion is at its lower bound of 1 and marginal costs are constant.
- Then, from the average marginal cost equation (12), we have

$$\frac{W_t^n}{P_t} = \mathcal{M}^{-1}(1-\alpha)A_t(N_t^n)^{-\alpha}.$$

The flexible price equilibrium III

- Use this with the labour supply condition to get:

$$\varphi_0 \frac{(N_t^n)^{\varphi}}{(C_t^n)^{-\sigma}} = \frac{W_t^n}{P_t} = \mathcal{M}^{-1}(1-\alpha)A_t(N_t^n)^{-\alpha}.$$

- The aggregate resource constraint will give us:

$$C_t^n = Y_t^n = A_t(N_t^n)^{1-\alpha}.$$

- These equations will allow us to price the flexible price – or “natural” – level of output This implies that the flexible price output is:

$$Y_t^n = \left(\frac{1-\alpha}{\varphi_0 \mathcal{M}}\right)^{\frac{1-\alpha}{\sigma(1-\alpha)+\alpha+\varphi}} A_t^{\frac{1+\varphi}{\sigma(1-\alpha)+\alpha+\varphi}}. \tag{32}$$

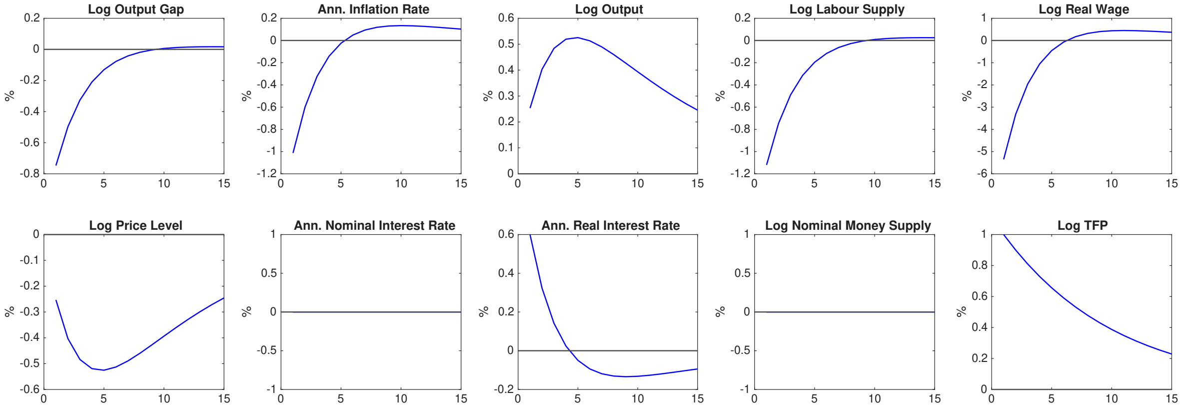
The flexible price equilibrium IV

- Note that if $\sigma = 1$, then N_t^n is a constant and not a function of A_t.
- In other words, if prices are flexible and $\sigma = 1$ (meaning we have log utility), labour hours would not react to technology shocks A_t.
- What is driving this is that, if $\sigma = 1$, then preferences are consistent with King, Plosser, and Rebelo (1988) preferences in which the income and substitution effects of changes in A_t exactly offset.
- Also note that in the flex price equilibrium, nominal shocks have no real effects. This makes sense as we no longer have any nominal rigidities or stickiness.

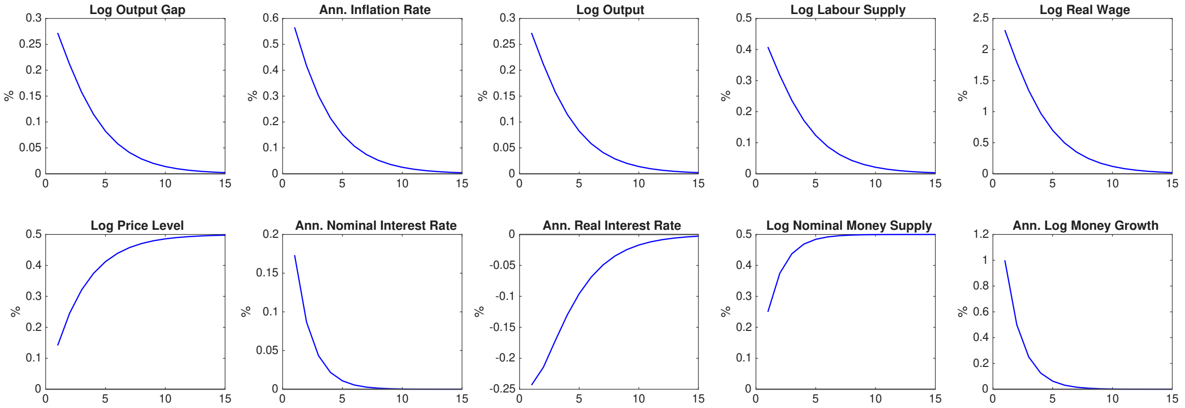
Quantitative analysis

- We can solve the model quantitatively in Dynare using a first order approximation about the steady state.
- I calibrate the model using the parameter values as in Galí (2015).
- Also, full credit to Dynare extraordinaire Johannes Pfeifer (who is also a great macroeconomist!) for doing the original replication – I've plugged his fantastic work already, but check out his `GitHub` for a collection of replication files he has done.

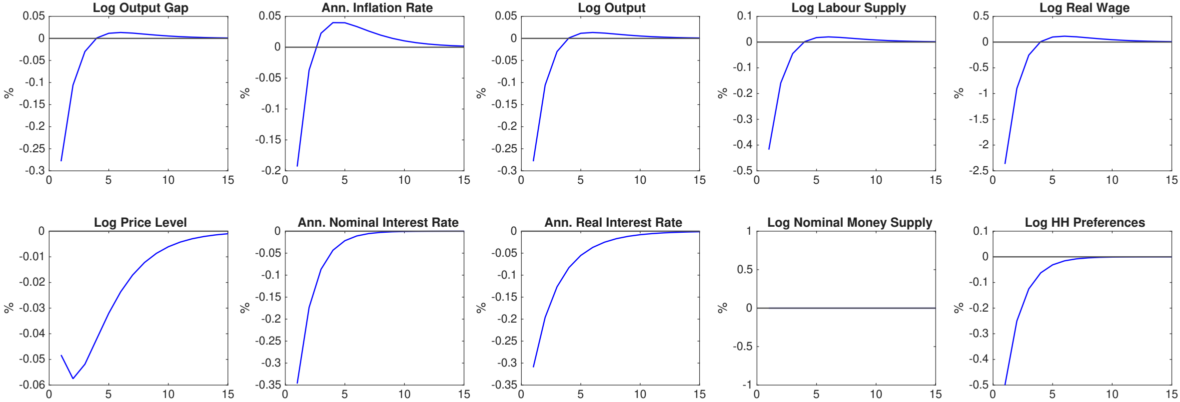
IRFs to a TFP shock



IRFs to a monetary shock (money supply rule)



IRFs to a preference shock



Log-Linearising the New Keynesian Model

Log-linearisation: Euler equation I

- Things we know about the steady state: $\pi = 0$, $P = P^* \implies MC = \mathcal{M}^{-1}$.
- Then with the Euler equation, use $C_t = Y_t$ to write:

$$C_t^{-\sigma} = \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma} R_t P_t}{P_{t+1}} \frac{Z_{t+1}}{Z_t}$$
$$\Leftrightarrow Y_t^{-\sigma} = \beta \mathbb{E}_t \frac{Y_{t+1}^{-\sigma} R_t P_t}{P_{t+1}} \frac{Z_{t+1}}{Z_t}$$
$$\Leftrightarrow Y_t^{-\sigma} = \beta \mathbb{E}_t \frac{Y_{t+1}^{-\sigma} R_t}{\Pi_{t+1}} \frac{Z_{t+1}}{Z_t},$$

and then take logs:

$$\begin{aligned} -\sigma \ln Y_t &= \ln \beta - \sigma \mathbb{E}_t \ln Y_{t+1} + \ln R_t - \mathbb{E}_t \ln \Pi_{t+1} + \ln Z_{t+1} - \ln Z_t \\ \Leftrightarrow -\sigma y_t &= \ln \beta - \sigma \mathbb{E}_t y_{t+1} + i_t - \mathbb{E}_t \pi_{t+1} + (\rho_z - 1) z_t, \end{aligned} \tag{33}$$

Log-linearisation: Euler equation II

then use our log-linear rules (I use a Taylor series expansion about the steady state):

$$-\sigma y - \frac{\sigma}{Y}(Y_t - Y) = \ln\beta - \sigma y - \frac{\sigma}{Y}(\mathbb{E}_t Y_{t+1} - Y) + i + (i_t - i)$$
$$- \pi - (\mathbb{E}_t \pi_{t+1} - \pi) + (\rho_z - 1)z + (\rho_z - 1)(z_t - z),$$

where the $-\sigma y$ terms cancel out:

$$-\frac{\sigma}{Y}(Y_t - Y) = \ln\beta - \frac{\sigma}{Y}(\mathbb{E}_t Y_{t+1} - Y) + i + (i_t - i) - \pi - (\mathbb{E}_t \pi_{t+1} - \pi) + (\rho_z - 1)z + (\rho_z - 1)(z_t - z),$$

and since $\pi = 0$ we know $i = \rho$.

- Since we're taking logs $\ln R_t = i_t$, and we know $1 + \rho = \frac{1}{\beta}$, hence $\ln(1 + \rho) = -\ln\beta$, and so we can write $-\ln\beta = i$.

Log-linearisation: Euler equation III

- We also have that $z = 0$ in the steady state. Thus, we have

$$-\frac{\sigma}{Y}(Y_t - Y) = -\frac{\sigma}{Y}(\mathbb{E}_t Y_{t+1} - Y) + (i_t - i) + (\mathbb{E}_t \pi_{t+1} - \pi) + (\rho_z - 1) z_t$$

$$\Leftrightarrow -\sigma \hat{y}_t = -\sigma \mathbb{E}_t \hat{y}_{t+1} + \hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} + (\rho_z - 1)\hat{z}_t,$$

where $\hat{y}_t = y_t - y \approx \frac{Y_t - Y}{Y}$ denotes percent (log) deviations from steady state for Y_t, and variables already in rate form are expressed as absolute deviations (e.g. $\hat{\pi}_t = \pi_t - \pi$ and $\hat{i}_t = i_t - i$). We can rewrite the above equation as:

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \sigma^{-1}\left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) + \sigma^{-1}(1 - \rho_z)\hat{z}_t, \tag{34}$$

which is called the “New Keynesian IS Curve” or “Dynamic IS Equation” (DISE).

Log-linearisation: Euler equation IV

- Alternatively – or equivalently, rather – we could take (33) and write the DISE in log-levels as Galí (2015) does:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1} - \rho) + \frac{1}{\sigma}(1 - \rho_z) z_t. \tag{35}$$

You can see that (34) and (35) are equivalent; the former is just approximated about the steady state given by the Fisher equation, $i = \rho$.

Log-linearisation: Labour supply I

- Now, let's look at the labour supply equation:[1]

$$\varphi_0 N_t^{\varphi} = C_t^{-\sigma} \frac{W_t}{P_t},$$
$$\implies \ln \varphi_0 + \varphi n_t = w_t - p_t - \sigma c_t,$$

but we know that:

$$mc_t = w_t - p_t - \ln(1-\alpha) + \frac{\alpha}{1-\alpha} y_t - \frac{1}{1-\alpha} a_t$$
$$\Leftrightarrow w_t - p_t = mc_t - \frac{\alpha}{1-\alpha} y_t - \frac{1}{1-\alpha} a_t + \ln(1-\alpha)$$

and since $c_t = y_t$, we can write the labour supply condition as by substituting for $w_t - p_t$:

$$\varphi n_t = mc_t - \frac{[\alpha + \sigma(1-\alpha)]}{1-\alpha} y_t + \frac{1}{1-\alpha} a_t + \ln(1-\alpha) - \ln \varphi_0.$$

Log-linearisation: Labour supply II

- Note that if we subtract φn from both the LHS and RHS, we could write everything in terms of log-deviations from steady state:

$$\varphi \hat{n}_t = \hat{mc}_t - \frac{[\alpha + \sigma(1-\alpha)]}{1-\alpha}\hat{y}_t + \frac{1}{1-\alpha}\hat{a}_t. \tag{36}$$

Log-linearisation: Price dispersion I

- Next, we take logs of the production function:

$$y_t = a_t + (1-\alpha)n_t - v_t^P.$$

- What is v_t^P? This is going to be messy... First, start by taking logs of the price dispersion equation:

$$V_t^P = \left[(1-\theta)(\Pi_t^*)^{-\frac{\epsilon}{1-\alpha}}\Pi_t^{\frac{\epsilon}{1-\alpha}} + \theta\Pi_t^{\frac{\epsilon}{1-\alpha}}(V_{t-1}^P)^{\frac{1}{1-\alpha}}\right]^{1-\alpha}$$

$$\ln V_t^P \equiv v_t^P = (1-\alpha)\ln\left[(1-\theta)(\Pi_t^*)^{-\frac{\epsilon}{1-\alpha}}\Pi_t^{\frac{\epsilon}{1-\alpha}} + \theta\Pi_t^{\frac{\epsilon}{1-\alpha}}(V_{t-1}^P)^{\frac{1}{1-\alpha}}\right]$$

Log-linearisation: Price dispersion II

Now, totally differentiate the above to get:

$$v_t^P - v^P = \frac{(1-\alpha)}{V^P}\left\{\begin{array}{c} -\frac{\epsilon}{1-\alpha}(1-\theta)(\Pi^*)^{-\frac{\epsilon}{1-\alpha}-1}\Pi^{\frac{\epsilon}{1-\alpha}}(\pi_t^* - \pi^*) \\ +\frac{\epsilon}{1-\alpha}(1-\theta)(\Pi^*)^{-\frac{\epsilon}{1-\alpha}}\Pi^{\frac{\epsilon}{1-\alpha}-1}(\pi_t - \pi) \\ +\frac{\epsilon}{1-\alpha}\theta\Pi^{\frac{\epsilon}{1-\alpha}-1}V^P(\pi_t - \pi) + \frac{1}{1-\alpha}\theta\Pi^{\frac{\epsilon}{1-\alpha}}(V^P)^{\frac{1}{1-\alpha}-1}(V_{t-1}^P - V^P) \end{array}\right\},$$

simplify things by using our facts about $V^P = 1$ and $\pi = \pi^* = 0$:

$$v_t^P = \frac{(1-\alpha)}{V^P}\left[-\frac{\epsilon}{1-\alpha}(1-\theta)\pi_t^* + \frac{\epsilon}{1-\alpha}(1-\theta)\pi_t + \frac{\epsilon}{1-\alpha}\theta V^P\pi_t + \frac{\theta}{1-\alpha}(V_{t-1}^P - V^P)\right]$$

$$\implies v_t^P = -\epsilon(1-\theta)\pi_t^* + \epsilon(1-\theta)\pi_t + \epsilon\theta\pi_t + \theta v_{t-1}^P,$$

and this can be written as:

$$v_t^P = -\epsilon(1-\theta)\pi_t^* + \epsilon\pi_t + \theta v_{t-1}^P. \tag{37}$$

Log-linearisation: Price dispersion III

- Next, log-linearise the equation for the evolution of inflation:

$$\begin{aligned} \Pi_t^{1-\epsilon} &= (1-\theta)(\Pi_t^*)^{1-\epsilon} + \theta \\ \implies (1-\epsilon)\ln \Pi_t &= \ln\left[(1-\theta)(\Pi_t^*)^{1-\epsilon} + \theta\right] \\ \therefore (1-\epsilon)\pi_t &= \ln\left[(1-\theta)(\Pi_t^*)^{1-\epsilon} + \theta\right], \end{aligned}$$

and then totally differentiate:

$$(1-\epsilon)(\pi_t - \pi) = \Pi^{\epsilon-1}\left[(1-\epsilon)(1-\theta)(\Pi^*)^{-\epsilon}(\pi_t^* - \pi^*)\right],$$

where $\Pi^{\epsilon-1}$ shows up because the term inside the large square parentheses is equal to $\Pi^{1-\epsilon}$ in the steady state, and when we take the derivative of the log this term gets inverted at the steady state.

Log-linearisation: Price dispersion IV

- We can use what we know about the zero inflation steady state to write:

$$\begin{aligned}(1-\epsilon)\pi_t &= (1-\epsilon)(1-\theta)\pi_t^* \\ \Leftrightarrow \pi_t &= (1-\theta)\pi_t^*. \end{aligned} \tag{38}$$

- In words, actual inflation is just proportional to reset price inflation, where the constant is equal to the fraction of firms that are updating their prices.
- Now use this by substituting it into the expression for price dispersion (37) to get:

$$\begin{aligned} v_t^P &= -\epsilon(1-\theta)\pi_t^* + \epsilon(1-\theta)\pi_t + \epsilon\theta\pi_t + \theta v_{t-1}^P \\ \therefore v_t^P &= \theta v_{t-1}^P. \end{aligned} \tag{39}$$

Log-linearisation: Price dispersion V

- This is a fairly important equation to note. If we are approximating about the zero inflation steady state where $V^P = 1$ or $v^P = 0$, then we're starting from a position in which $v^P_{t-1} = 0$, which means that $v^P_t = 0, \forall t$.
- In other words, about a zero inflation steady state, price dispersion is a second order phenomenon, and we can just ignore it up to a first order approximation about a zero inflation steady state.

Log-linearisation: Production I

- We now move onto log-linearising the aggregate output identity.
- As we just showed above, in the vicinity of the steady state $v_t^P = 0$, and so we can write:

$$y_t = a_t + (1-\alpha)n_t. \tag{40}$$

- Then, substitute $n_t = (y_t - a_t)/(1-\alpha)$ into the labour supply condition (36) to get

$$\varphi\left(\frac{y_t - a_t}{1-\alpha}\right) = mc_t - \frac{[\alpha + \sigma(1-\alpha)]}{1-\alpha}y_t + \frac{1}{1-\alpha}a_t + \ln(1-\alpha) - \ln\varphi_0$$

$$\implies mc_t = \varphi\left(\frac{y_t - a_t}{1-\alpha}\right) + \frac{\alpha + \sigma(1-\alpha)}{1-\alpha}y_t - \frac{1}{1-\alpha}a_t - \ln(1-\alpha) + \ln\varphi_0$$

$$mc_t = \frac{\sigma(1-\alpha) + \alpha + \varphi}{1-\alpha}y_t - \frac{(1+\varphi)}{1-\alpha}a_t - \ln(1-\alpha) + \ln\varphi_0, \tag{41}$$

Log-linearisation: Production II

where just for completeness I used the log-level version of (36). Now, we know that

$$Y_t^n = \left(\frac{1-\alpha}{\varphi_0 \mathcal{M}}\right)^{\frac{1-\alpha}{\sigma(1-\alpha)+\alpha+\varphi}} A_t^{\frac{1+\varphi}{\sigma(1-\alpha)+\alpha+\varphi}},$$

and log-linearising this we have

$$y_t^n = \left(\frac{1-\alpha}{\sigma(1-\sigma)+\alpha+\varphi}\right)[\ln(1-\alpha) - \ln\varphi_0 - \ln\mathcal{M}] + \left(\frac{1+\varphi}{\sigma(1-\alpha)+\alpha+\varphi}\right)a_t. \tag{42}$$

Log-linearisation: Production III

- But then notice that we can use this expression to write:

$$\left(\frac{1-\alpha}{\sigma(1-\alpha)+\alpha+\varphi}\right)\ln\mathcal{M} = -y_t^n + \left(\frac{1-\alpha}{\sigma(1-\alpha)+\alpha\varphi}\right)[\ln(1-\alpha)-\ln\varphi_0]$$
$$+\left(\frac{1+\varphi}{\sigma(1-\alpha)+\alpha+\varphi}\right)a_t$$
$$\mu_t \equiv \ln\mathcal{M} = -\left(\frac{\sigma(1-\alpha)+\alpha+\varphi}{1-\alpha}\right)y_t^n + \frac{1+\varphi}{1-\alpha}a_t + \ln(1-\alpha) - \ln\varphi_0 \tag{43}$$
$$\Leftrightarrow \mu_t = p_t - \psi_t,$$

where μ_t is the [log] average price markup and $\psi_t = \ln\Psi_t$ is the log nominal marginal cost.

Log-linearisation: Production IV

- I'm trying to map what we have done here to Galí (2015). So, just to be clear:

$$mc_t = \psi_t - p_t = -\mu_t.$$

- When prices are flexible ($\theta = 0$), the average markup is constant and equal to the "desired markup" μ, so (43) evaluated at the flexible price equilibrium is:

$$\mu = -\left(\frac{\sigma(1-\alpha)+\alpha+\varphi}{1-\alpha}\right) y_t^n + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \ln(1-\alpha) - \ln\varphi_0. \tag{44}$$

Log-linearisation: Production V

- Then we can use this expression to write y_t^n compactly as:

$$y_t^n = \psi_{ya} a_t + \psi_y, \tag{45}$$

where

$$\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\alpha+\varphi}, \quad \psi_y \equiv -\frac{(1-\alpha)\left[\mu + \ln \varphi_0 - \ln(1-\alpha)\right]}{\sigma(1-\alpha)+\alpha+\varphi}.$$

- Alright, hopefully you're still following – I've done all this derivation because I want to be explicit about the different ways of writing these expressions, and also so that we can follow the notation of Galí (2015).
- Having said that, I will deviate slightly from what Galí does and instead of using μ_t I will use the expression for the real marginal cost (41).

Log-linearisation: Production VI

- Substitute for a_t using the expression for the flexible price level of output ((42) or (45)) to write:

$$
\begin{aligned}
mc_t &= \frac{\sigma(1-\alpha)+\alpha+\varphi}{1-\alpha} y_t - \left(\frac{1+\varphi}{1-\alpha}\right)\left(\frac{y_t^n - \psi_y}{\psi_{ya}}\right) - \ln(1-\alpha) + \ln \varphi_0 \\
&= \frac{\sigma(1-\alpha)+\alpha+\varphi}{1-\alpha}(y_t - y_t^n) + \mu, \\
\implies \widehat{mc}_t &= \frac{\sigma(1-\alpha)+\alpha+\varphi}{1-\alpha}\tilde{y}_t. \qquad (46)
\end{aligned}
$$

- In words, deviations of real marginal cost are proportional to the output gap, $\tilde{y}_t \equiv y_t - y_t^n$.
- Recall that real marginal cost is the inverse of the price markup in steady state.

Log-linearisation: Production VII

- So if the gap is zero, then markups are equal to the desired fixed steady state markup of $\mathcal{M} = \frac{\epsilon}{\epsilon - 1}$.
- If the output gap is positive, then real marginal cost is above its steady state, so markups are lower than desired (equivalently, the economy is less distorted).
- The converse is true when the gap is negative. So, equivalently, as in Galí (2015):

$$\hat{\mu}_t = -\left(\frac{\sigma(1-\alpha)+\alpha+\varphi}{1-\alpha}\right)\tilde{y}_t. \tag{47}$$

Log-linearisation: the NKPC I

- We now need to log-linearise the auxiliary variables:

$$\frac{X_{1,t}}{P_t^{\epsilon+b}} = C_t^{-\sigma} MC_t Y_t + \theta\beta\mathbb{E}_t \Pi_{t+1}^{\epsilon+b} \frac{X_{1,t+1}}{P_{t+1}^{\epsilon+b}},$$

$$\frac{X_{2,t}}{P_t^{\epsilon-1}} = C_t^{-\sigma} Y_t + \theta\beta\mathbb{E}_t \Pi_{t+1}^{\epsilon-1} \frac{X_{2,t+1}}{P_{t+1}^{\epsilon+b}}.$$

- This is going to be messy... Let's start with $X_{1,t}$. I'm actually going to just make *another* pair of auxiliary variables because it'll make things a lot easier. Let's define

$$\tilde{X}_{1,t} \equiv \frac{X_{1,t}}{P_t^{\epsilon+b}} = C_t^{-\sigma} MC_t Y_t + \theta\beta\mathbb{E}_t \Pi_{t+1}^{\epsilon+b} \tilde{X}_{1,t+1}, \tag{48}$$

$$\tilde{X}_{2,t} \equiv \frac{X_{2,t}}{P_t^{\epsilon-1}} = C_t^{-\sigma} Y_t + \theta\beta\mathbb{E}_t \Pi_{t+1}^{\epsilon-1} \tilde{X}_{2,t+1}. \tag{49}$$

Log-linearisation: the NKPC II

- Then, because we have $Y_t = C_t$, we can write the following:

$$\tilde{x}_{1,t} = \ln\left[Y_t^{1-\sigma} MC_t + \theta\beta\mathbb{E}_t\Pi_{t+1}^{\epsilon+b}\tilde{X}_{1,t+1}\right]$$

and totally differentiating we have:

$$\begin{aligned}
\tilde{x}_{1,t} - \tilde{x}_1 &\approx \frac{1}{\tilde{X}_1}\left\{\begin{array}{c} Y^{1-\sigma}(MC_t - MC) + (1-\sigma)MCY^{-\sigma}(Y_t - Y) \\ +(\epsilon+b)\theta\beta\Pi^{\epsilon+b-1}\tilde{X}_1(\mathbb{E}_t\pi_{t+1} - \pi) + \theta\beta\mathbb{E}_t\Pi^{\epsilon+b}(\mathbb{E}_t\tilde{X}_{1,t+1} - \tilde{X}_1) \end{array}\right\} \\
\hat{\tilde{x}}_{1,t} &= \frac{Y^{1-\sigma}}{\tilde{X}_1}(MC_t - MC) + \frac{(1-\sigma)Y^{-\sigma}MC}{\tilde{X}_1}(Y_t - Y) \\
&\quad + \frac{(\epsilon+b)\theta\beta\Pi^{\epsilon+b-1}\tilde{X}_1}{\tilde{X}_1}(\mathbb{E}_t\pi_{t+1} - \pi) + \frac{\theta\beta\Pi^{\epsilon+b}}{\tilde{X}_1}(\mathbb{E}_t\tilde{X}_{1,t+1} - X_1) \\
&= \frac{Y^{1-\sigma}MC}{\tilde{X}_1}\widehat{mc}_t + \frac{(1-\sigma)Y^{1-\sigma}MC}{\tilde{X}_1}\hat{y}_t + (\epsilon+b)\theta\beta\mathbb{E}_t\pi_{t+1} + \theta\beta\mathbb{E}_t\hat{\tilde{x}}_{1,t+1},
\end{aligned}$$

Log-linearisation: the NKPC III

and in the steady state we know that $\tilde{X}_1 = Y^{1-\sigma}MC/(1-\theta\beta)$, which yields

$$\hat{\tilde{x}}_{1,t} = (1-\theta\beta)\widehat{mc}_t + (1-\sigma)(1-\theta\beta)\hat{y}_t + (\epsilon + b)\theta\beta\mathbb{E}_t\pi_{t+1} + \theta\beta\hat{\tilde{x}}_{1,t+1}. \tag{50}$$

- Now we can deal with $\tilde{x}_{2,t}$. Start by taking logs:

$$\tilde{x}_{2,t} = \ln\left[Y_t^{1-\sigma} + \theta\beta\mathbb{E}_t\Pi_{t+1}^{\epsilon-1}\tilde{X}_{2,t+1}\right],$$

Log-linearisation: the NKPC IV

and then totally differentiate:

$$\tilde{x}_{2,t} - \tilde{x}_2 \approx \frac{1}{\tilde{X}_2} \left\{ \begin{array}{c} (1-\sigma)Y^{-\sigma}(Y_t - Y) + (\epsilon - 2)\theta\beta\Pi^{\epsilon-2}\tilde{X}_2(\mathbb{E}_t\pi_{t+1} - \pi) \\ +\theta\beta\Pi^{\epsilon-1}(\mathbb{E}_t\tilde{X}_{2,t+1} - \tilde{X}_2) \end{array} \right\}$$

$$\hat{\tilde{x}}_{2,t} = \frac{(1-\sigma)Y^{-\sigma}(Y_t - Y)}{\tilde{X}_2} + (\epsilon-1)\theta\beta\Pi^{\epsilon-2}(\mathbb{E}_t\pi_{t+1} - \pi) + \frac{\theta\beta\Pi^{\epsilon-1}(\mathbb{E}_t\tilde{X}_{2,t+1} - \tilde{X}_2)}{\tilde{X}_2}$$

$$= \frac{(1-\sigma)Y^{1-\sigma}}{\tilde{X}_2}\hat{y}_t + (\epsilon-1)\theta\beta\pi_t + \theta\beta\mathbb{E}_t\hat{\tilde{x}}_{2,t+1},$$

and we know $\tilde{x}_2 = \frac{Y^{1-\sigma}}{1-\theta\beta}$, so we have:

$$\hat{\tilde{x}}_{2,t} = (1-\sigma)(1-\theta\beta)\hat{y}_t + (\epsilon-1)\theta\beta\mathbb{E}_t\pi_{t+1} + \theta\beta\mathbb{E}_t\hat{\tilde{x}}_{2,t+1}. \tag{51}$$

Log-linearisation: the NKPC V

- Subtracting $\hat{\hat{x}}_{2,t}$ from $\hat{\hat{x}}_{1,t}$ yields:

$$
\begin{aligned}
\hat{\hat{x}}_{1,t} - \hat{\hat{x}}_{2,t} = (1-\theta\beta)\widehat{mc}_t + (1-\sigma)(1-\theta\beta)\hat{y}_t + (\epsilon + b)\theta\beta\mathbb{E}_t\pi_{t+1} + \theta\beta\mathbb{E}_t\hat{\hat{x}}_{1,t+1} \\
- (1-\sigma)(1-\theta\beta)\hat{y}_t - (\epsilon - 1)\theta\beta\mathbb{E}_t\pi_{t+1} - \theta\beta\mathbb{E}_t\hat{\hat{x}}_{2,t+1} \\
= (1-\theta\beta)\widehat{mc}_t + \theta\beta(1+b)\mathbb{E}_t\pi_{t+1} + \theta\beta(\mathbb{E}_t\hat{\hat{x}}_{1,t+1} - \mathbb{E}_t\hat{\hat{x}}_{2,t+1}).
\end{aligned}
$$

Log-linearisation: the NKPC VI

- Next we log-linearise the reset price expression in order to find $\hat{\tilde{x}}_{1,t} - \hat{\tilde{x}}_{2,t}$:

$$
\begin{aligned}
(\Pi_t^*)^{1+b} &= \mathcal{M}\frac{X_{1,t}/P_t^{\epsilon+b}}{X_{2,t}/P_t^{\epsilon-1}}\Pi_t^{1+b}, \\
(\Pi_t^*)^{1+b} &= \mathcal{M}\frac{\tilde{X}_{1,t}}{\tilde{X}_{2,t}}\Pi_t^{1+b}, \\
\implies (1+b)\pi_t^* &= \mu + \tilde{x}_{1,t} - \tilde{x}_{2,t} + (1+b)\pi_t \\
\pi_t^* &= \frac{\mu + \tilde{x}_{1,t} - \tilde{x}_{2,t}}{1+b} + \pi_t \\
\Leftrightarrow \tilde{x}_{1,t} - \tilde{x}_{2,t} &= (1+b)(\pi_t^* - \pi_t) - \mu.
\end{aligned}
$$

Log-linearisation: the NKPC VII

- But from (38) we have:

$$\pi_t^* = \frac{1}{1-\theta}\pi_t,$$

and so

$$\tilde{x}_{1,t} - \tilde{x}_{2,t} = \frac{(1+b)\theta}{1-\theta}\pi_t - \mu.$$

- Putting things together, we can then write:

$$\hat{\tilde{x}}_{1,t} - \hat{\tilde{x}}_{2,t} = \frac{(1+b)\theta}{1-\theta}\pi_t = (1-\theta\beta)\widehat{mc}_t + \theta\beta(1+b)\mathbb{E}_t\pi_{t+1} + \theta\beta\frac{(1+b)\theta}{1-\theta}\mathbb{E}_t\pi_{t+1},$$

and with a bit of cleaning up:

$$\begin{aligned} \pi_t &= \frac{(1-\theta)(1-\theta\beta)}{(1+b)\theta}\widehat{mc}_t + \beta\mathbb{E}_t\pi_{t+1} \\ \Leftrightarrow \pi_t &= \lambda\widehat{mc}_t + \beta\mathbb{E}_t\pi_{t+1} \\ \Leftrightarrow \pi_t &= \beta\mathbb{E}_t\pi_{t+1} - \lambda\hat{\mu}_t, \end{aligned} \tag{52}$$

Log-linearisation: the NKPC VIII

where

$$\lambda \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta} \ominus$$

- Expression (52) is called the "New Keynesian Phillips Curve" (NKPC). It is "new" because it is forward-looking unlike classic Phillips Curves, but it's a Phillips Curve in the sense that it captures a relationship between inflation and some real measure.
- We can re-write the NKPC in terms of the output gap by using (46):

$$\pi_t = \lambda \frac{(\sigma(1-\alpha)+\alpha+\varphi)}{1-\alpha}\tilde{y}_t + \beta \mathbb{E}_t \pi_{t+1}$$
$$\Leftrightarrow \pi_t = \kappa \tilde{y}_t + \beta \mathbb{E}_t \pi_{t+1}, \tag{53}$$

where $\kappa = \lambda(\sigma(1-\alpha)+\alpha+\varphi)/(1-\alpha)$ and is often referred to as the slope of the NKPC.

Log-linearisation: Housekeeping I

- The rest of the equations are fairly straightforward. The expressions for a_t, z_t, and money growth are already log-linear so we have:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \tag{54}$$
$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \tag{55}$$
$$\Delta m_t = \Delta(m_t - p_t) + \pi_t \tag{56}$$
$$\Delta(m_t - p_t) + \pi_t = \rho_m \Delta(m_{t-1} - p_{t-1}) + \rho_m \pi_{t-1} + \varepsilon_t^m. \tag{57}$$

- Then, just take logs of the money demand function to linearise it:

$$m_t - p_t = y_t - \eta i_t,$$

Which is pretty straightforward: Demand for real money balances is decreasing in the real interest rate and increasing in y_t (think of the LM curve).

Log-linearisation: Housekeeping II

- Finally, there's one more thing we need to do: write the DISE (35) in terms of the output gap:

$$y_t - y_t^n - \mathbb{E}_t y_{t+1}^n = \mathbb{E}_t(y_{t+1} - y_{t+1}^n) - y_t^n - \frac{1}{\sigma}(i_t - \mathbb{E}_t\pi_{t+1} - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t$$

$$\tilde{y}_t - (\psi_{ya}\mathbb{E}_t a_{t+1} + \psi_y) = \mathbb{E}_t\tilde{y}_{t+1} - (\psi_{ya}a_t + \psi_y) - \frac{1}{\sigma}(i_t - \mathbb{E}_t\pi_{t+1} - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t$$

$$\tilde{y}_t = \mathbb{E}_t\tilde{y}_{t+1} + (1-\rho_a)\psi_{ya}a_t - \frac{1}{\sigma}(i_t - \mathbb{E}_t\pi_{t+1} - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t.$$

Do a bit of cleaning up:

$$\tilde{y} = \mathbb{E}_t\tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n), \tag{58}$$

Log-linearisation: Housekeeping III

where r_t^n is the natural rate of interest – the real interest rate that would prevail in the flexible price equilibrium also known as the “Wicksellian natural rate of interest”:

$$r_t^n = \rho - \sigma(1 - \rho_a)\psi_{ya}a_t + (1 - \rho_z)z_t. \tag{59}$$

- Trivia: Wicksell’s most influential contribution was his theory of interest, originally published in German as *Geldzins und Gíterpreise*, in 1898. The English translation *Interest and Prices* became available in 1936; a literal translation of the original title would read *Money Interest and Commodity Prices*. Wicksell invented the key term natural rate of interest and defined it at that interest rate which is compatible with a stable price level.

Log-linearised equations

- The complete log-linearised system of equations is:

$$
\begin{aligned}
\tilde{y}_t &= \mathbb{E}_t\tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n),\\
\pi_t &= \beta\mathbb{E}_t\pi_{t+1} + \kappa\tilde{y}_t,\\
r_t^n &= \rho - \sigma(1-\rho_a)\psi_{ya}a_t + (1-\rho_z)z_t,\\
\Delta m_t &= \Delta(m_t - p_t) + \pi_t,\\
\Delta(m_t - p_t) + \pi_t &= \rho\Delta(m_{t-1} - p_{t-1}) + \rho_m\pi_{t-1} + \varepsilon_t^m,\\
m_t - p_t &= y_t - \eta i_t,\\
a_t &= \rho_a a_{t-1} + \varepsilon_t^a,\\
z_t &= \rho_z z_{t-1} + \varepsilon_t^z.
\end{aligned}
$$

- But we can go simpler...

Implementing a Taylor Rule

NK model with a Taylor rule I

- So far, our model has contained an exogenous rule for money growth.
- But this doesn't seem to match how monetary policy is conducted.
- We want monetary policy to focus on changing the interest rate in response to endogenous changes in inflation and output.
- A popular interest rate rule is the Taylor rule:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t, \tag{60}$$

- where $\widehat{y} = y_t - y$; and ϕ_π, ϕ_y, and ρ_i are coefficients, with $\phi_\pi > 1$.
- v_t is an exogenous monetary policy shock that evolves according to the AR(1) process:

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v.$$

NK model with a Taylor rule II

- Taylor (1993)'s original rule was

$$i_t^F = 4 + 1.5(\pi_t - 2) + 0.5(y_t - y_t^*),$$

  where i_t^F is the Federal Funds Rate, π_t is annual inflation, and y_t^* was trend (log) GDP.
- Notice that we could rewrite (60) in terms of the output gap:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_y \widehat{y}_t^n + v_t,$$

  where $\widehat{y}_t^n \equiv y_t^n - y$. Notice that money does not enter the interest rate rule.
- We can replace the money growth process with the interest rate rule and assume that the central bank provides sufficient money at all times to meet money demand at the interest rate.

NK model with a Taylor rule III

- Thus, our log-linearised New Keynesian model with a Taylor rule can be compactly written as:

$$\text{DISE: } \tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right), \tag{61}$$

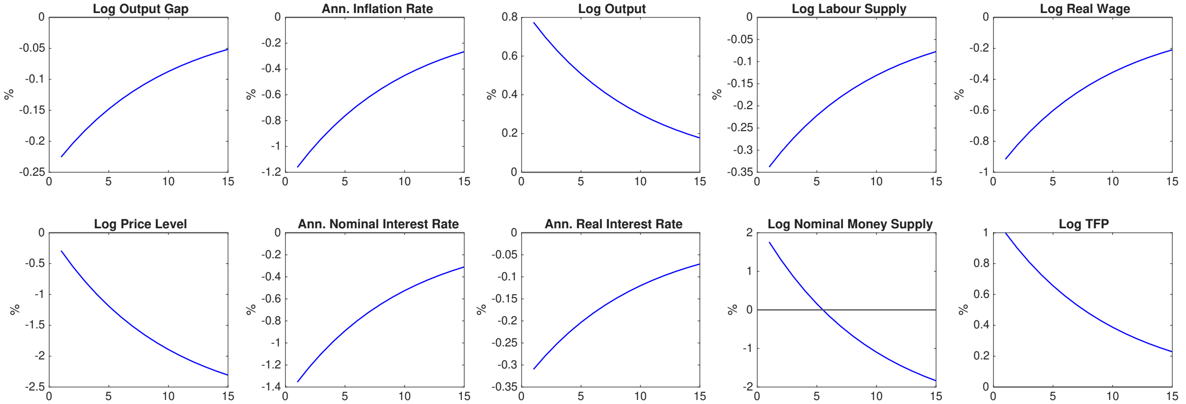
$$\text{NKPC: } \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t, \tag{62}$$

$$\text{Taylor Rule: } i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t, \tag{63}$$

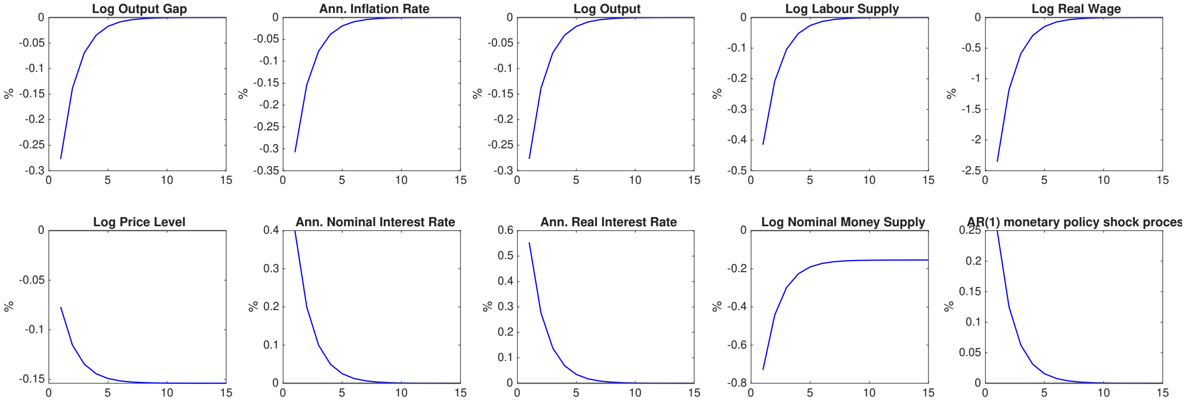
$$\text{Natural Rate: } r_t^n = \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t. \tag{64}$$

  This is referred to in the macroeconomic literature as the "canonical New Keynesian model", often written down with just the first three equations, with the law of motion for r_t^n in the background (since it is driven by exogenous shocks).

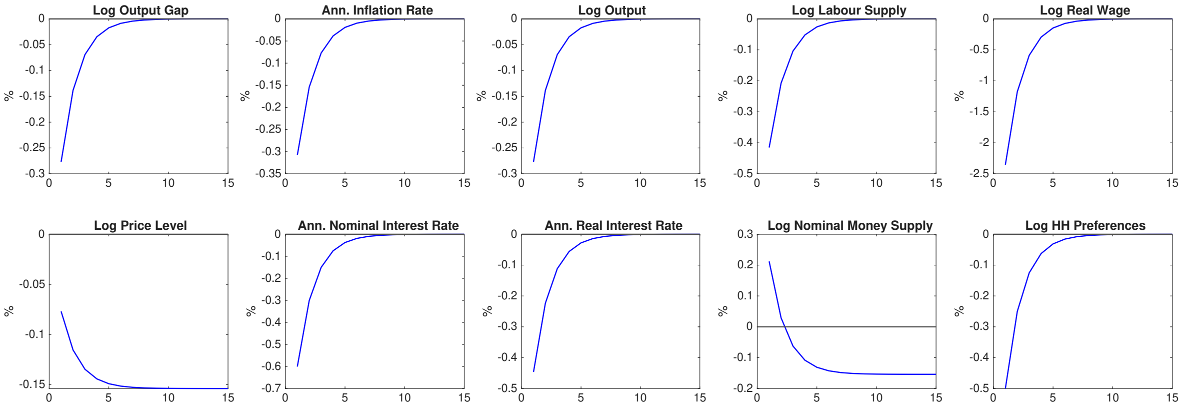
IRFs to a TFP shock



IRFs to a monetary shock (money supply rule)



IRFs to a preference shock



Method of Undetermined Coefficients

The Rational Expectations solution

- The three equation New Keynesian (NK) model has two jump/control variables $(\pi_t, \tilde{y}_t)$ and an exogenous state variable, r_t^n.
- We could solve for the policy functions mapping the states into the jump variables – `Dynare` will do this very easily for us.
- Or, since we don't have capital and investment in this model, we can use the method of undetermined coefficients.
- The method of undetermined coefficients involves us guessing a policy function function (linear in this case since the system is log-linear), imposing that, and the solving a system of equations for the policy rule coefficients.
- In a small scale model without capital, this is pretty easy to do, and gives us nice analytical solutions.

Simple example: Shocks with no persistence I

- Consider the three equation NK model with the exogenous process for r_t^n:

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right),$$
$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t,$$
$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_y \widehat{y}_t^n + v_t,$$
$$r_t^n = \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t.$$

- Assume $z_t = v_t = 0, \forall t$, and $\rho_a = 0$.
- Express things as log deviations from steady-sate (to get rid of constants).
- Use definition of natural output (45) to replace y_t^n in the Taylor rule.

Simple example: Shocks with no persistence II

- So, we have:

$$\begin{aligned} \tilde{y}_t &= \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - \hat{r}_t^n\right), \\ \pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t, \\ \hat{i}_t &= \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_y \psi_{ya} \varepsilon_t^a, \\ \hat{r}_t^n &= -\sigma \psi_{ya} \varepsilon_t^a. \end{aligned}$$

- Since i_t and r_t^n are not state variables, we can substitute them out of the DISE:

$$\begin{aligned} \tilde{y}_t &= \mathbb{E}_t \tilde{y}_{t+1} - \sigma^{-1}\left[\phi_\pi \pi_t + \phi_y \tilde{y}_t + \psi_{ya}(\phi_y + \sigma)\varepsilon_t^a\right], \\ \pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t. \end{aligned}$$

Simple example: Shocks with no persistence III

- Now, we want to look at how our jump variables react to our shocks. First, guess that the policy functions look like:

$$\tilde{y}_t = A_y \varepsilon_t^a,$$
$$\pi_t = A_\pi \varepsilon_t^a,$$

and then plug them into the DISE and the NKPC:

$$A_y \varepsilon_t^a = A_y \mathbb{E}_t \varepsilon_{t+1}^a - \sigma^{-1} \left[\phi_\pi A_\pi + \phi_y A_y + \psi_{ya}(\phi_y + \sigma) \right] \varepsilon_t^a, \quad (65)$$
$$A_\pi \varepsilon_t^a = \beta A_\pi \mathbb{E}_t \varepsilon_{t+1}^a + \kappa A_y \varepsilon_t^a. \quad (66)$$

Simple example: Shocks with no persistence IV

- Then, since the shocks are assumed to be IID with mean zero, we can simplify this down to:

$$A_y = -\sigma^{-1}\left[\phi_\pi A_\pi + \phi_y A_y + \psi_{ya}(\phi_y + \sigma)\right],$$
$$A_\pi = \kappa A_y.$$

- Then we plug A_π back into the guessed DISE:

$$A_y = -\sigma^{-1}\left[\phi_\pi \kappa A_y + \phi_y A_y + \psi_{ya}(\phi_y + \sigma)\right]$$
$$A_y = -\frac{\psi_{ya}(\phi_y + \sigma)}{\sigma + \phi_\pi \kappa + \phi_y}$$

Simple example: Shocks with no persistence V

- So, our policy functions are

$$\tilde{y}_t = -\frac{\psi_{ya}(\phi_y + \sigma)}{\sigma + \phi_\pi \kappa + \phi_y} \varepsilon_t^a, \tag{67}$$

$$\pi_t = -\frac{\psi_{ya}(\phi_y + \sigma)\kappa}{\sigma + \phi_\pi \kappa + \phi_y} \varepsilon_t^a. \tag{68}$$

Example: Multiple shocks with persistence I

- Again, suppose we have the DISE, NKPC, Taylor rule, and a law of motion for the natural interest rate:

$$
\begin{aligned}
\tilde{y}_t &= \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right), \\
\pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t, \\
i_t &= \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_y \widehat{y}_t^n + v_t, \\
r_t^n &= \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t.
\end{aligned}
$$

Example: Multiple shocks with persistence II

- Combine these together to get:

$$\tilde{y}_t = \mathbb{E}_t\tilde{y}_{t+1} - \frac{1}{\sigma}\left[\underbrace{\rho + \phi_\pi\pi_t + \phi_y\tilde{y}_t + \phi_y\hat{y}_t^n + v_t}_{i_t} - \mathbb{E}_t\pi_{t+1}\underbrace{-\rho + \sigma(1-\rho_a)\psi_{ya}a_t - (1-\rho_z)z_t}_{-r_t^n}\right],$$

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\tilde{y}_t.$$

- Do some rearranging and cleaning up to isolate the period t and $t+1$ jump variables, and define an auxiliary variable $u_t \equiv \hat{r}_t^n - \phi_y\hat{y}_t^n - v_t$:[2]

$$\left(\sigma + \phi_y\right)\tilde{y}_t + \phi_\pi\pi_t = \sigma\mathbb{E}_t\tilde{y}_{t+1} + \mathbb{E}_t\pi_{t+1}\underbrace{-\psi_{ya}(\phi_y + \sigma(1-\rho_a))a_t + (1-\rho_z)z_t - \phi_y\hat{y}_t^n - v_t}_{u_t},$$

$$-\kappa\tilde{y}_t + \pi_t = \beta\mathbb{E}_t\pi_{t+1}.$$

Example: Multiple shocks with persistence III

- This can be written as:

$$\underbrace{\begin{bmatrix} \sigma + \phi_y & \phi_\pi \\ -\kappa & 1 \end{bmatrix}}_{\boldsymbol{A}_0} \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \underbrace{\begin{bmatrix} \sigma & 1 \\ 0 & \beta \end{bmatrix}}_{\boldsymbol{A}_1} \begin{bmatrix} \mathbb{E}_t \tilde{y}_{t+1} \\ \mathbb{E}_t \pi_{t+1} \end{bmatrix} + \tilde{\boldsymbol{B}} u_t.$$

- We need to then invert $\boldsymbol{A}_0$ in order to write the system as:

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \boldsymbol{A}_T \begin{bmatrix} \mathbb{E}_t \tilde{y}_{t+1} \\ \mathbb{E}_t \pi_{t+1} \end{bmatrix} + \boldsymbol{B}_T u_t. \tag{69}$$

Example: Multiple shocks with persistence IV

- We can do this matrix inversion very easily in MATLAB:

```
syms sigma phi_y phi_pi kappa rho beta;
A0 = [sigma + phi_y, phi_pi; -kappa, 1];
A1 = [sigma, 1; 0, beta];
B = [1; 0];
det_A0 = det(A0); % Compute the determinant of A0
A0_inv = inv(A0); % Compute the inverse of A0
A0_inv_A1 = simplify(A0_inv * A1); % Compute A0^{-1} A1
A0_inv_B = simplify(A0_inv * B); % Compute A0^{-1} B
% Display the results
disp('A0^{-1} A1:');
disp(A0_inv_A1);
disp('A0^{-1} B:');
disp(A0_inv_B);
```

Example: Multiple shocks with persistence V

- So, we get:

$$\boldsymbol{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa+\beta(\sigma+\phi_y) \end{bmatrix}, \quad \boldsymbol{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix},$$

  where $\Omega \equiv (\sigma + \phi_y + \kappa\phi_\pi)^{-1}$.
- Now we can use the method of undetermined coefficients to solve for the model, and see how $\tilde{y}_t$ and π_t respond to shocks.
- Assume that u_t follows a stationary AR(1) process with an autoregressive coefficient $\rho_u \in [0, 1)$.
- Again, propose a guess for the policy function:

$$\tilde{y}_t = A_y u_t,$$
$$\pi_t = A_\pi u_t.$$

Example: Multiple shocks with persistence VI

- Plugging these guesses into (69), and use the fact that $\mathbb{E}_t u_{t+1} = \rho_u u_t$ to get:

$$A_y = (1 - \beta\rho_u)\Lambda_u, \tag{70}$$

$$A_\pi = \kappa\Lambda_u, \tag{71}$$

where

$$\Lambda_u \equiv \frac{1}{(1 - \beta\rho_u)\left[\sigma(1 - \rho_u) + \phi_y\right] + \kappa(\phi_\pi - \rho_u)},$$

and where $\Lambda_u > 0$ if the Taylor Principle holds.

An Alternative to Calvo: Rotemberg Pricing

Rotemberg pricing I

- Under Rotemberg pricing, all intermediate goods firms are able to adjust their pricing, but with a quadratic adjustment cost.
- In equilibrium they all behave identically, since the adjustment costs are identical across all firms, which makes aggregation work out nicely.
- Whether a NKPC is derived via Calvo or Rotemberg pricing makes little difference up to a first order approximation about a zero inflation steady state (Ascari and Rossi, 2012).
- Intermediate firms still face the same demands from final goods firms under Rotemberg pricing and they produce output according to:

$$Y_t(j) = A_t N_t(j)^{1-\alpha}.$$

Rotemberg pricing II

- An intermediate firm j's cost minimisation problem is:

$$\min_{N_t(j)} \; W_t N_t(j),$$

subject to :

$$A_t N_t(j)^{1-\alpha} \geq Y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\epsilon} Y_t.$$

Rotemberg pricing III

- So the Lagrangian for the firm problem is:

$$\mathcal{L} = W_t N_t(j) - \Psi_t(j) \left\{ A_t N_t(j)^{1-\alpha} - \left[\frac{P_t(j)}{P_t} \right]^{-\epsilon} Y_t \right\},$$

which yields the following FOC:

$$\begin{aligned} \mathcal{L}_{N_t(j)} : \; W_t &= (1-\alpha)\Psi_t(j) A_t N_t(j)^{-\alpha} \\ \implies \Psi_t(j) &= \frac{W_t}{(1-\alpha) A_t N_t(j)^{-\alpha}}, \end{aligned} \tag{72}$$

which is exactly what we had in (11)!

Rotemberg pricing IV

- The real marginal cost is also the same as before:

$$MC_t(j) = \frac{\Psi_t(j)}{P_t} = \frac{W_t/P_t}{(1-\alpha)A_t N_t(j)^{-\alpha}}$$

- I mentioned before that all firms behave the same under Rotemberg pricing, and so we could simply drop the j index. But let's keep it for now and formulate the profit maximisation problem for firm j.
- The nominal flow profit for producer j is given by:

$$D_t(j) = P_t(j)Y_t(j) - W_t N_t(j) - \frac{\psi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t, \qquad (73)$$

where ψ is the Rotemberg cost of price adjustment parameter, and is measured in units of the final good.

Rotemberg pricing V

- Next, write the profit function in real terms:

$$\frac{D_t(j)}{P_t} = \frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t}{P_t} N_t(j) - \frac{\psi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t,$$

and since $W_t/P_t = (1-\alpha)MC_t A_t N_t(j)^{-\alpha}$:

$$\frac{D_t(j)}{P_t} = \frac{P_t(j)}{P_t} Y_t(j) - (1-\alpha)MC_t \underbrace{A_t N_t(j)^{-\alpha} N_t(j)}_{Y_t(j)} - \frac{\psi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t,$$

and then sub in for $Y_t(j)$:

$$\begin{aligned}\frac{D_t(j)}{P_t} &= \frac{P_t(j)}{P_t}\left[\frac{P_t(j)}{P_t}\right]^{-\epsilon} Y_t - (1-\alpha)MC_t\left[\frac{P_t(j)}{P_t}\right]^{-\epsilon} Y_t - \frac{\psi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t \\ &= \left[\frac{P_t(j)}{P_t}\right]^{1-\epsilon} Y_t - (1-\alpha)MC_t\left[\frac{P_t(j)}{P_t}\right]^{-\epsilon} Y_t - \frac{\psi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t.\end{aligned}$$

Rotemberg pricing VI

- So firms choose $P_t(j)$ to maximise the expected present discounted value of flow profit each period, where discounting is done by the household's stochastic discount factor:

$$\frac{\partial \frac{D_t(j)}{P_t}}{\partial P_t(j)} = (1-\epsilon)\left[\frac{P_t(j)}{P_t}\right]^{-\epsilon}\frac{Y_t}{P_t} + \epsilon(1-\alpha)MC_t\left[\frac{P_t(j)}{P_t}\right]^{\epsilon-1}\frac{Y_t}{P_t} - \psi\left(\frac{P_t(j)}{P_{t-1}(j)}-1\right)\frac{Y_t}{P_{t-1}(j)}$$
$$+\psi\mathbb{E}_t M_{t,t+1}\left(\frac{P_{t+1}(j)}{P_t(j)}-1\right)\frac{P_{t+1}(j)}{P_t(j)}\frac{Y_{t+1}}{P_t(j)} = 0, \qquad (74)$$

Rotemberg pricing VII

and rearrange:

$$(\epsilon - 1)\left[\frac{P_t(j)}{P_t}\right]^{-\epsilon}\frac{Y_t}{P_t} = \epsilon(1-\alpha)MC_t\left[\frac{P_t(j)}{P_t}\right]^{\epsilon-1}\frac{Y_t}{P_t} - \psi\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)\frac{Y_t}{P_{t-1}(j)}$$
$$+ \psi\mathbb{E}_t M_{t,t+1}\left(\frac{P_{t+1}(j)}{P_t(j)} - 1\right)\left[\frac{P_{t+1}(j)}{P_t(j)}\right]\frac{Y_{t+1}}{P_t(j)},$$

then divide both the LHS and RHS by Y_t, multiply both the LHS and RHS by P_t, and note that gross inflation $\Pi_t = \frac{P_t}{P_{t-1}}$, and since all firms behave identically, $P_t(j) = P_t$:

$$\epsilon - 1 = \epsilon(1-\alpha)MC_t - \psi\left(\frac{P_t}{P_{t-1}} - 1\right)\frac{P_t}{P_{t-1}} + \psi\mathbb{E}_t M_{t,t+1}\left(\frac{P_{t+1}}{P_t} - 1\right)\frac{P_{t+1}}{P_t}\frac{Y_{t+1}}{P_t}\frac{P_t}{Y_t}$$
$$= \epsilon(1-\alpha)MC_t - \psi(\Pi_t - 1)\Pi_t + \psi\mathbb{E}_t M_{t,t+1}(\Pi_{t+1} - 1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}.$$

Rotemberg pricing VIII

- Phew! Now, we need to log-linearise!

$$\ln(\epsilon - 1) = \ln\left\{\epsilon(1-\alpha)MC_t - \psi(\Pi_t - 1)\Pi_t + \psi\mathbb{E}_t M_{t,t+1}(\Pi_{t+1} - 1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right\},$$

then totally differentiate[3]

$$0 = \frac{1}{\epsilon - 1}\left\{\epsilon(1-\alpha)dMC_t - \psi d\Pi_t + \psi\beta\mathbb{E}_t d\Pi_{t+1}\right\},$$

and since we know that in the steady state $\pi = 0$ and $d\Pi_t = \pi_t$:

$$0 = \frac{\epsilon(1-\alpha)}{\epsilon - 1}dMC_t - \frac{\psi}{\epsilon - 1}\pi_t + \frac{\beta\psi}{\epsilon - 1}\mathbb{E}_t\pi_{t+1}.$$

Rotemberg pricing IX

- Now, we need to use a little trick. We know that $\frac{\epsilon}{\epsilon-1} = \mathcal{M}$ is nothing but MC^{-1}, so the first term on the RHS is:

$$\frac{(1-\alpha)dMC_t}{MC} = (1-\alpha)\frac{MC_t - MC}{MC} = (1-\alpha)\widehat{mc}_t,$$

so we have:

$$\begin{aligned} 0 &= (1-\alpha)\widehat{mc}_t - \frac{\psi}{\epsilon-1}\pi_t + \frac{\beta\psi}{\epsilon-1}\mathbb{E}_t\pi_{t+1}, \\ \implies \frac{\psi}{\epsilon-1}\pi_t &= (1-\alpha)\widehat{mc}_t + \frac{\beta\psi}{\epsilon-1}\mathbb{E}_t\pi_{t+1} \\ \therefore \pi_t &= \frac{(\epsilon-1)(1-\alpha)}{\psi}\widehat{mc}_t + \beta\mathbb{E}_t\pi_{t+1}, \end{aligned} \tag{75}$$

Rotemberg pricing X

which is nothing but the NKPC, and it will match the Calvo pricing-based NKPC if:

$$\psi = \frac{(\epsilon - 1)(1-\alpha)\theta}{(1-\theta)(1-\theta\beta)\Theta}.$$

- Finally, under Rotemberg pricing, the aggregate resource constraint comes out to:

$$Y_t = C_t + \frac{\psi}{2}(\Pi_t - 1)^2 Y_t, \tag{76}$$

Rotemberg pricing XI

and log-linearising this yields:

$$\ln Y_t = \ln \left[C_t + \frac{\psi}{2}(\Pi_t - 1)^2 Y_t \right]$$

$$dY_t = dC_t + \frac{\psi}{2} 2(\Pi - 1) Y d\pi_t + \frac{\psi}{2}(\Pi - 1)^2 dY_t$$

$$\frac{dY_t}{Y} = \frac{dC_t}{Y} + \psi(\Pi - 1) d\pi_t + \frac{\frac{\psi}{2}(\Pi - 1)^2 dY_t}{Y},$$

$$\hat{y}_t = \frac{dC_t}{Y},$$

but $Y = C$, so:

$$\therefore \hat{y}_t = \hat{c}_t. \tag{77}$$

Conclusion I

- The “canonical three-equation” New Keynesian model we covered in these lectures is based on a log-linearisation about the zero inflation steady state.
- This has kept much of analysis very simple. But those interested in trend inflation should refer to Ascari (2004) and Ascari and Ropele (2009).
- There are also alternatives to the Calvo and Rotemberg sticky price models we worked with here: see, for example, Chari, Kehoe, and McGrattan (2000) and models of sticky information (Mankiw and Reis, 2002).
- As Galí points out, the empirical performance of the NKPC has always been controversial. Many studies have criticised the NKPC for its poor performance. See, for example, Mavroeidis, Plagborg-Møller, and Stock (2014).
- But in recent years, a number of contributions have found a close relationship between the empirical relevance of the NKPC when it is adjusted to account for labour markets (Benigno and Eggertsson, 2023, 2024; Siena and Zago, 2024).

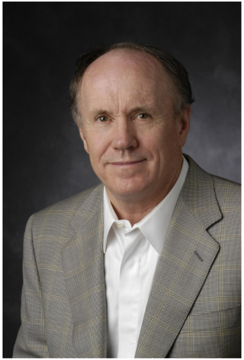
Conclusion II

- But despite the many flaws of the NKPC and the New Keynesian DSGE framework more broadly, they have become workhorses of policy design, forecasting, and analysis in academia and especially in central banks.
- These DSGE models – in particular, the medium-scale New Keynesian models such as the Smets-Wouters model – have a lot of strengths: They fit the data well and allow us to answer many “what if” questions regarding policy and economic shocks.
- However, they have a list of weaknesses:
  - A large number of ad-hoc economic mechanisms designed mainly to fit persistence properties of the data rather than because economists have a strong belief in these particular stories;
  - A large amount of unexplained shocks which are often highly persistent;
  - A minimal treatment of banking and financial markets (still true despite current ongoing work);
  - Very limited modelling of policy tools or details of national accounts;
  - Plenty of evidence that pure Rational Expectations assumption is flawed; and

Conclusion III

* Claims that they are based on stable structural parameters and thus immune to the Lucas Critique are silly, and would most likely upset these two:

Robert E. Lucas Jr.



Edward C. Prescott

Conclusion IV

- But, they would disagree:

(a) Olivier J. Blanchard

(b) Jordi Galí

(c) Mark Gertler

References I

Ascari, Guido. 2004. "Staggered Prices and Trend Inflation: Some Nuisances"." *Review of Economic Dynamics* 7 (3): 642–667.

Ascari, Guido, and Tiziano Ropele. 2009. "Trend Inflation, the Taylor Principle and Determinacy." *Journal of Money, Credit and Banking* 41 (8): 1557–1584.

Ascari, Guido, and Lorenza Rossi. 2012. "Trend Inflation and Firms Price-Setting: Rotemberg verses Calvo." *The Economic Journal* 122 (563): 1115–1141.

Benigno, Pierpaolo, and Gauti B Eggertsson. 2023. *It's Baaack: The Surge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve.* Working Paper, Working Paper Series 31197. National Bureau of Economic Research. https://doi.org/10.3386/w31197. http://www.nber.org/papers/w31197.

References II

Benigno, Pierpaolo, and Gauti B Eggertsson. 2024. *The Slanted-L Phillips Curve.* Working Paper, Working Paper Series 32172. National Bureau of Economic Research. https://doi.org/10.3386/w32172. http://www.nber.org/papers/w32172.

Blanchard, Olivier J., and Nobuhiro Kiyotaki. 1987. "Monopolistic Compeitition and the Effects of Aggregate Demand." *American Economic Review* 77 (4): 647–666.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan. 2000. "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?" *Econometrica* 68 (5): 1151–1179. https://doi.org/https://doi.org/10.1111/1468-0262.00154.

Galí, Jordi. 2015. *Monetary Policy, Inflation, and the Business Cycle.* 2nd Edition. Princeton University Press.

References III

Goodfriend, Marvin, and Robert G. King. 1997. “The New Neoclassical Synthesis and the Role of Monetary Policy.” In *NBER Macroeconomics Annual 1997, Volume 12,* 231–296. National Bureau of Economic Research, Inc.

King, Robert G., Charles I. Plosser, and Sergio T. Rebelo. 1988. “Production, Growth, and Business Cycles I: The Basic Neoclassical Model.” *Journal of Monetary Economics* 21:195–232.

Mankiw, Gregory N., and Ricardo Reis. 2002. “Sticky Information Verses Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve.” *The Quarterly Journal of Economics* 117 (4): 1295–1328.

Mavroeidis, Sophocles, Mikkel Plagborg-Møller, and James H. Stock. 2014. “Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve.” *Journal of Economic Literature* 52 (1): 124–88. https://doi.org/10.1257/jel.52.1.124.

References IV

Siena, Daniele, and Riccardo Zago. 2024. "Job Polarisation, Labour Market Fluidity and the Flattening of the Phillips Curve." *The Economic Journal,* ISSN: 0013-0133. https://doi.org/10.1093/ej/ueae006.

Taylor, John B. 1993. "Discretion Verses Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy,* 195–214.

Woodford, Michael. 2003. *Interest and Prices.* Princeton University Press.